NDU

MAT 235

Ordinary Differential Equations

Final Exam

Duration: 2 hours

Name: _____

Section: _____

Instructor: _____

Grade: _____

1) (12 points) Solve

a) $\left(\frac{y}{x}+\tan\frac{y}{x}\right)dx-dy=0$

b) $\frac{dy}{dx}=\frac{y}{x}-\frac{5}{2}x^2y^3$

2) (10 points) Solve $y'' - 2y' + y = \dfrac{e^x}{1+x^2}$.

3) **(15 points)** Find the Laplace transform $F(s)$ of each of the following functions $f(t)$.

a) $f(t)=\int_0^t y\sin y dy$ b) $f(t)=\int_0^t (t-y)^5.\cos y dy$ c) $f(t)=\frac{e^{2t}-1}{t}$

(Recall that, if the Laplace Transform of $f(t)=F(s)$, then the Laplace transform of $\frac{f(t)}{t}=\int_s^\infty f(y)dy$).

4) **(15 points)** Find the inverse Laplace Transform $f(t)$ of each of the following functions $F(s)$.

a) $F(s)=\dfrac{2s-1}{s^2+2s+1}$ b) $F(s)=\arctan\dfrac{1}{s}$ c) $F(s)=\ln\left(\dfrac{s^2+1}{s^2}\right)$

5) (10 points) Use the Laplace transform to solve

$y'' - 2y' + y = 4e^{-t}$ with $y(0) = 2$, and $y'(0) = -1$.

6) (10 points) Solve

$$\left.\begin{aligned} x' - x - y &= 2e^{+} \\ 4x - y' + y &= 0 \end{aligned}\right\} \text{ with } x(0) = 0, \text{ and } y(0) = 2$$

7) (18 points) Use the method of Frobenius to find the general solution near $x = 0$ of $xy'' + 2y' + xy = 0$.

8) **(10 points)** Find the generalized power series solution near $x = 0$ of $(x^2 + 1)y'' - 2xy' + 2y = 0$ (Hint: the point $x = 0$ is an ordinary point of the given equation).