

NDU

MAT 235

Ordinary Differential Equations

Final Exam

Duration: 2 hours

Name: _____

Section: _____

Instructor: _____

Grade: _____

1) (12 points) Solve the following differential equations:

a) $x^2 dy + (xy - xy^3) dx = 0$.

b) $(y^3 + x^2 y) dx - (xy^2 + x^3) dy = 0$

2) (10 points) Solve $y'' + 4y = \sec 2x$.

- 3) (8 points)** Find the orthogonal trajectories to the family of curves $x^2 = 2y - 1 + Ce^{-2y}$ where C is an arbitrary constant.

4) (6 points) Use the convolution to find the Laplace inverse of: $G(s) = \frac{1}{s^2(s-1)}$.

5) (10 points) Solve the following initial-value problem using Laplace transform.

$$y'' + y' = \begin{cases} 0 & 0 \leq t < 1 \\ e^t & t \geq 1 \end{cases} \quad y(0) = y'(0) = 0$$

6) (10 points) Solve the differential system by the Laplace transform method:

$$x' = x - 2y; \quad y' = 4x + 5y$$

$$x(0) = 1, \quad y(0) = -2$$

7) (10 points) Use the elimination method to solve the linear system:

$$x' - 6x + 3y = 8e^t$$

$$y' - 2x - y = 4e^t$$

- 8) (14 points)** Given that $x_0 = 0$ is an ordinary point, find the general solution of the differential equation $(x^2 + 1)y'' - 2xy' + 2y = 0$.

- 9) (20 points)** Show that $x_0 = 0$ is a regular singular point of the differential equation $4x^2 y'' - 8x^2 y' + (4x^2 + 1)y = 0$, then use the Frobenius method to find the general solution.

