NDU

Notre Dame University

MAT 235

Ordinary Differential Equations

Final Exam

Duration: 2 hours

Name:

Section:

Instructor:

Grade:

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1) (10 points) Find the family of orthogonal trajectories of the family of curves $y = c \ln x$, for $x > 0$.

2) (20 points) Solve the following differential equations.

a) $(x-y)dx-(x+y)dy=0$

b) $y' + \frac{y}{x} = x^2 y^3$

3) (20 points) Solve the following differential equations.

a) $x^2 y'' - xy' = -2x^4, \quad x > 0$

b) $y''' - y' = 2e^x$

4) (8 points) Consider the initial-value problem

$$y'' + x^2 y = 0, \qquad y(0) = 1, \quad y'(0) = 0$$

Find its power series solution near the ordinary point $x_0 = 0$.

5) (18 points) Consider the differential equation

$$x^2 y'' + (-x + x^2) y' - 3(x+1) y = 0$$

a) Show that $x_0 = 0$ is a regular singular point.

b) Find the indicial equation.

c) Find two linearly independent solutions.

d) Deduce the general solution of the differential equation.

6) (14 points) Solve the following initial-value problem using Laplace transform.

$$y'' + y' = \begin{cases} e^t & 0 \le t < 1 \\ 0 & 1 \le t \end{cases}, \quad y(0) = y'(0) = 0$$

7) (10 points) Solve the following linear system.

$$y_1' = 2y_1 + y_2$$
$$y_2' = y_1 + 2y_2$$