NDU

MAT 235

Ordinary Differential Equations

Final Exam

Duration: 2 hours

Name: _____

Section: A

Instructor: Dr. Ishac Zoghbi

Grade: _____

1) **(10 points)** Solve the following differential equation.

$2xyy' + (x-1)y^2 = x^2e^x$. (***Hint:*** Use the substitution $v = y^2$).

2) **(10 points)** Solve $y'' + 4y' + 4y = x^2 e^{-2x}$.

3) (6 points)

a) Find the Laplace transform of $t\sin t$.

b) Deduce the value of the function $f(t)$, which verifies the following equation.

$2\int_0^t f(u)f'(t-u)du = t\sin t$ with $f(0)=0$.

4) **(6 points)** Find Laplace transform of $\frac{1-e^t}{t}$.

5) **(6 points)** Use convolution product to find the Laplace inverse of $\frac{6}{s^4(s+1)}$.

6) **(10 points)** Use Lapalce transform to solve the following initial-value problem.

$$\begin{aligned} y_1' &= 4y_1 - 2y_2 \\ y_2' &= 5y_1 + 2y_2 \end{aligned}; \quad y_1(0) = 2, \quad y_2(0) = -2$$

7) **(6 points)** Solve $y'' + 2y' + y = \delta(t-1);\ y(0) = 1,\ y'(0) = -1.$

8) **(10 points)** Use Laplace transform to solve

$$y'' + 6y' - 7y = \begin{cases} t, & \text{if } 0 \le t < 1 \\ 0, & \text{if } t \ge 1 \end{cases}; \; y(0) = y'(0) = 0$$

9) **(20 points)** Show that $x_0 = 0$ is a regular singular point of the differential equation $xy'' + (1-2x)y' + (x-1)y = 0$, then use the Frobenius method to find the general solution .

10) (16 points) Use the eigenvalue – eigenvector method to solve the following system.

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$