NDU

MAT 235

Ordinary Differential Equations

Final Exam

Duration: 120 Minutes.

Name: _____

Section: _____

Instructor: _____

Grade: _____

1- (9 points) Solve each of the differential equations:

a) $\left(\sin y + x^2 + 2x\right)dx + \cos y\, dy = 0$

b) $3\frac{dy}{dx}+\frac{2}{x+1}y=\frac{x^3}{y^2}$

2- (5 points) Given $f(t)=\int_0^t (t-u)^5 \sin u \, du,$ find Laplace transform of $f(x)$

3- (7 points) Given $F(s)=\dfrac{s+2}{(s+2)^2+9}e^{\frac{-\pi s}{2}}$, find Laplace inverse of $F(s)$

4- (10 points) Find the general solution of the differential equation
$y'' + 4y' + 4y = x^{-4}e^{-2x}$ for $x > 0$

5- (8 points) Find the general solution of the system

$$x' = -4x - y$$

$$y' = x - 2y$$

6- (7 points) Find the general solution of $(x+1)^2 y'' - 2(x+1)y' + 2y = 0$. For $x+1>0$ (Hint: you can use the substitution $u = x+1$) .

7- (12 points) Use Laplace transforms to solve the following initial-value problem

$$y'' + y = h(t) \text{ where } h(t) = \begin{cases} t & 0 < t < \pi \\ 0 & t \geq \pi \end{cases} \text{ with } y(0) = 2,\ y'(0) = 0$$

8- (6 points) Find a family of oblique trajectories that interest the family of hyperbolas $xy = c$ at angle 45^{o}

9- (16 points) Use the Laplace transform to find the solution of the given linear system that satisfies the given initial conditions

$$x' + x + y' + 2y = 0$$

$$7x' - 5x + 8y' - 4y = 0$$

with $x(0) = 1$ and $y(0) = -1$

(20 points) Given the differential equation $x^2 y'' + (x^2 - 3x)y' + (4 - 2x)y = 0.$ Show that $x_0 = 0$ is a regular singular point, then use the method of Frobenius to find solutions of the given equation in some interval $0 < x < R.$