

NDU

MAT 235

Ordinary Differential Equations

Final Exam

Wednesday February 4, 2004

Duration: 2 hours

Name: _____

Section: _____

Instructor: _____

Grade: _____

Please note that you have 10 questions and 12 pages

1) (10 points) Solve the following:

a) $xydx + (x^2 + y^2 + 2)dy = 0$ with $y(0) = 1$

b) $\frac{dy}{dx} = (2x + y + 3)^2 + (2x + y + 3) - 2$

2) (10 points) Find the orthogonal trajectories of the family of circles $(x - c)^2 + y^2 = c^2$.

3) (4 points) Find the Laplace inverse of the following function $F(s) = \ln \frac{s}{s-3}$

4) (8 points) Solve $(1+x)y'' + y' = 1$ for $x > -1$

5) (10 points) Solve $y'' + 9y = \sec 3x$

6) (8 points) Use Laplace transform to solve

$$y'' + 6y' - 7y = \begin{cases} 2 & 0 \leq t < 1 \\ 0 & t \geq 1 \end{cases}$$

with $y(0) = y'(0) = 0$

7) (10 points) Solve the following initial-value problem by using Laplace transform.

$$\begin{aligned} y_1' + y_1 + 2y_2 &= 0 \\ 3y_1 + 2y_2 + y_2' &= 0 \end{aligned} \quad \text{with } y_1(0) = 1 \text{ and } y_2(0) = 2$$

8) (8 points) Use the eigenvalue-eigenvector method to solve

$$y_1' = 2y_1 + 2y_2$$

$$y_2' = 7y_1 - 3y_2$$

9) (12 points) Given that $x_0 = 0$ is an ordinary point of the differential equation $(x^2 + 1)y'' - 2xy' + 2y = 0$. Find the general power series solution in powers of x .

10) (20 points) Given $4x^2y'' - 8x^2y' + (4x^2 + 1)y = 0$ for $x > 0$.

a) Show that $x_0 = 0$ is a regular singular point.

b) Find the indicial roots.

c) Find the generalized power series solution in powers of x .

