

NDU

MAT 235

Ordinary Differential Equations

Final Exam

Duration: 55 minutes

Name: _____

Section: _____

Instructor: _____

Grade: _____

1) (12 points) Solve the following differential equations.

a) $(x^2 - y^2)dx - (xy)dy = 0$

b) $y' + \frac{2}{x}y = xy^4$

- 2) (8 points)** Find the family of orthogonal trajectories of the family of curves $y = c \ln x$, for $x > 0$.

3) (5 points) Given $f(t) = \int_0^t v e^v \sin(t-v) dv$, find the Laplace transform $L\{f(t)\}$.

4) (9 points) Find the general solution of the following system of differential equations.

$$y_1' = y_2$$

$$y_2' = -4y_1 + t$$

5) (12 points) Solve the following differential equation: $x^2y'' - 3xy' + 4y = 4x^{-1}$, $x > 0$.

- 6) (6 points)** Given the following clairaut's differential equation: $y - xy' = y' - y' \ln|y'|$.
- a) Find a general solution.

- b) Find a singular solution.

7) (6 points) Solve the following differential equation $y^{(4)} + 4y'' + 4y = 0$.

8) (12 points) Use Laplace transform to find the solution of the system

$$\begin{aligned} y_1' - 4y_1 + 3y_2 &= 1 \\ y_2' - 2y_1 + 3y_2 &= 0 \end{aligned} \quad \text{with} \quad y_1(0) = y_2(0) = 0$$

(Use this Page to continue problem 8)

9) (12 points) Solve the following initial-value problem using Laplace transform.

$$y'' - 6y' - 7y = \begin{cases} 0 & t < 1 \\ 56 & t \geq 1 \end{cases} \quad \text{with} \quad \begin{cases} y(0) = 0 \\ y'(0) = 0 \end{cases}$$

(Use this Page to continue problem 9)

10) **(18 points)** Consider the differential equation $(x^2 - x)y'' + (3x - 1)y' + y = 0$.

a) Show that $x_0 = 0$ is a regular singular point.

b) Use Frobenius theorem to find two linearly independent solutions $y_1(x)$ and $y_2(x)$.

c) Find a general solution for the given differential equation.