Notre Dame University
Faculty of Natural and Applied Sciences
Department of Mathematics and Statistics

MAT 235
Ordinary Differential Equations

Exam # 1

Duration: 55 minutes

Name: _____

Section: _____

Instructor: _____

Grade: _____

Directions
1. Write neatly and clearly.
2. Do not use pencils unless for graphing.
3. Show all work.
4. Scientific calculators are allowed.
5. Turn off your mobile phones.

Please note that you have 6 questions and a total of 7 pages

1) **(20 points)** Solve the following equation.

$$(y\ln y - 2xy)dx + (x + y)dy = 0.$$

2) **(15 points)** Solve $xy'' - y' = 3x^2$, given that $y'(1) = 5$ and $y(1) = 7$.

3) (10 points) the equation $\dfrac{dy}{dx} + P(x)y = Q(x)y^n$, which is known as Bernoulli's equation, is linear when $n = 0$ or 1. Show that it can be reduced to linear equation for any other value of n by the change of variable $z = \dfrac{1}{y^{n-1}}$.

4) **(15 points)** Consider the Clairaut equation $y = xy' - \ln y'$ for $x > 0$.

    a) Find a general solution of this equation.
    b) Find a singular solution of this equation.

5) **(20 points)** Use an appropriate transformation to solve $(x + y + 1)dx - (x + y - 1)dy = 0$.

6) **(20 points)** Find the orthogonal trajectories of the family of circles $x^2 + y^2 = 2\,cx$.