MAT 235– Ordinary Differential Equations
1st Midterm

1) **(15 points)** Find the orthogonal trajectories of the family of curves $y = \dfrac{cx^2}{x+1}$.

2) (**15 points**) Solve the initial value problem $\dfrac{dy}{dx} = \dfrac{x}{y} - \dfrac{x}{1+y}$, $\quad y(0) = 1$.

3)   (**15 points**) Solve the equation $xy' = \sqrt{1 - x^2 y^2} - y$.

           Hint: Let $z = xy$

4) **(15 points)** Solve the equation $\dfrac{dy}{dx} = \dfrac{y}{x} - \dfrac{5x^2 y^3}{2}$.

(4)

5) **(20 points)** Find an integrating factor for the equation

$$xydx + \left(x^2 + 2y^2 + 2\right)dy = 0, \text{ then solve it.}$$

(5)

6) **(20 points)** a) Verify that $y = x$ is a solution of the Riccati equation

$$(*) \quad \frac{dy}{dx} = -y^2 + xy + 1$$

b) Write the general solution of $(*)$.

(6)