Mat 211 – Discrete Mathematics
Exam # 2

Solve the following problems

1) a- The sequence $f(n)$ is defined recursively as follows
$f(1) = 1, f(n) = 2f(n-1)$ for $n \geq 2$. Find a formula for $f(n)$
b- Give a recursive definition of the sequence $\{b_n\}$, $n=1,2,\ldots$
if $b_n = 4n - 2$.

(20 pts)

2) a- A particular brand of shirts comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex.
How many types of this shirt are made?
b- A multiple choice test contains 10 questions. There are 4 possible answers for each question. How many ways can a student answer the questions on the test if every question is answered?

(20 pts)

3) What is the minimum number of students, each of whom comes from one of 50 high schools, enrolled in a university to guarantee that there are at least 10 who come from the same high school?

(10 pts)

4) Suppose that a department contains 3 men and 4 women. How many ways are there to form a committee with 4 members if it must have the same number of men and women?

(10 pts)

5) What is the coefficient of $x^2 y^8$ in the expansion of $(2x + 3y)^{10}$?

(10 pts)

6) Let R= { (1,3), (1,4), (2,3), (2,4), (3,1), (3,4) } be a relation on the set {1,2,3,4}
a) Find a matrix representation of R ١٩٣٤.
b) Represent R by a directed graph
c) Find the reflexive closure of R
d) Find the symmetric closure of R

(20 pts)

7) How many relations are there on a set with n elements that are reflexive and symmetric?

(10 pts)

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1) a) $f(1) = 1$

$f(n) = 2f(n-1) \quad n \geq 2$.

$n=2$ and $n=3$ and $n=4$ and $n=5$

$f(2) = 2f(1) = 2 \Leftrightarrow 2^1$

$f(3) = 2f(2) = 4 \quad 2^2$

$f(4) = 2f(3) = 8. \quad 2^3$

$f(5) = 2f(4) = 16 \quad 2^4$.

$\vdots$

$\Rightarrow f(n) = 2^{n-1}$. (10)

b) $b_n = 4n - 2$.

$n=1$

$b_1 = 4 - 2 = 2$.

$n=2 \qquad b_2 = b_1 + 4$

$b_2 = 8 - 2 = 6$.

$n=3 \qquad b_3 = b_2 + 4$

$b_3 = 12 - 2 = 10$

$\Rightarrow b_n = b_{n-1} + 4$ (10)

2) 12 colors.

male $\Rightarrow$ each t-shirt must have a

female. (color, gender, size).

3 size.

task1 : colors $n_1 = 12$ $\Rightarrow$ multiplication rule

task2 ; gene $n_2 = 2$ $\Rightarrow$ 72 ways.

task3 : size $n_3 = 3$

(10)

b) 10 questios.

q_1 : 4 possible ans
q_2 : " " "
q_3 : " " "
⋮
q_{10} : 4 —

according to multiplication rule
$4^{10} = 1048576$ ways. 10

3) 50 high schools.
at least 10 come from the same high school.

Let N ~~boxes~~ is the nb of ~~high schools~~ studen.
K boxes is the nb of high schools

$$\left\lceil \frac{N}{K} \right\rceil = \left\lceil \frac{N}{50} \right\rceil = 10$$

$$9 < \frac{N}{50} \leq 10$$

10

$$450 < N \leq 500$$

$\Rightarrow$ the minimum nb of students is $N = 451$.

4) 3M   4W.

Committee with 4 members. if same
nb of Men and women
$\Rightarrow$ 2 Men and 2 Women.

$$\Rightarrow C(3,2) \times C(4,2)$$
$$= \frac{3!}{2!\,1!} \times \frac{4!}{2!\,2!} = 3 \times 6 = 18.$$

5) $(2x+3y)^{10} = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^{j}$ · coeff of x^2y^8.

$n=10$
$j=8$

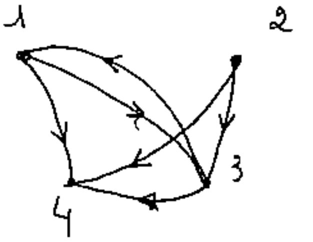
$$\binom{10}{8}(2x)^2(3y)^8$$

$$= \underbrace{\binom{10}{8} 2^2 \times 3^8}_{A} \left(x^2y^8\right)$$ 10

$\Rightarrow$
$A = 1180980$.

6) $R = \{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$.
$A = \{1,2,3,4\}$ $A = \{1,2,3,4\}$

a) $$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

b) 

c) reflexive closure of R

$R' = R \cup \Delta$

such that $\Delta = \{(1,1), (2,2), (3,3), (4,4)\}$. $(4,4)$.

$\Rightarrow R' = \{(1,1).(2,2)(3,3)(4,4)(1,3)(1,4)(2,3)(3,1)(3,4)\}$.

d) symmetric closure of R.

$R' = R \cup R^{-1}$

~~such that~~ $\cancel{R^{-1}}$ $R^{-1} = \{(3,1)(4,1)(3,2)(4,2)(1,3)(4,3)\}$.

so that $R' = \{(1,3)(3,1)(1,4)(4,1)(2,3)(3,2)$
$(2,4)(4,2)(3,4)(4,3)\}$.

7) $A = \{a_1, a_2, a_3, a_4, \dots a_n\}$
n element

$A \times A = n^2$ elements

total nb of relations is 2^{n^2}.

$$\begin{bmatrix} a_1b_1 & a_1b_2 & a_1b_3 & \dots & a_1b_n \\ a_2b_1 & a_2b_2 & \dots & & \\ & & a_3b_3 & & \\ & & & \ddots & \\ a_nb_1 & \dots & \dots & \dots & \end{bmatrix}$$

2

R is reflexive nb of ~~sets~~ R that are reflexive is 2^n (n = nb of diagonal elements).

R, is symmetric. nb of elements $N = (n)(n-1) \times \dots (n-r+1)$.
~~which is equal to~~ $\Rightarrow$ nb of subsets is 2^N.

and Now $R' = R \cup R_1$ $2^n \times 2^N$ relation R'