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1) **(15 points)** Show, by two different methods, that if A, B and C are sets, then

$$A-(B\cap C)=(A-B)\cup(A-C)$$

1st method: Truth table

A	B	C	$B\cap C$	$A-(B\cap C)$	$A-B$	$A-C$	$(A-B)\cup(A-C)$
T	T	T	T	F	F	F	F
T	T	F	F	T	F	T	T
T	F	T	F	T	T	F	T
T	F	F	F	T	T	T	T
F	T	T	T	F	F	F	F
F	T	F	F	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

Use 0's and 1's

2nd method:

$A-(B\cap C)\subseteq(A-B)\cup(A-C)$

Let $k\in A-(B\cap C)\Rightarrow k\in A\wedge k\notin B\cap C$

$\Rightarrow k\in A\wedge k\in\overline{B\cap C}$

$\Rightarrow k\in A\wedge k\in\bar{B}\cup\bar{C}$

$\Rightarrow k\in A\wedge(k\in\bar{B}\vee k\in\bar{C})$

$\Rightarrow(k\in A\wedge k\notin B)\vee(k\in A\wedge k\notin C)$

$\Rightarrow k\in(A-B)\cup(A-C)$

true

$(A-B)\cup(A-C)\subseteq A-(B\cap C)$

Let $k\in(A-B)\cup(A-C)\Rightarrow(k\in A\wedge k\notin B)\vee(k\in A\wedge k\notin C)$

$\Rightarrow(k\in A\wedge k\in\bar{B})\vee(k\in A\wedge k\in\bar{C})$

$\Rightarrow(k\in A\vee k\in A)\wedge(k\in A\vee k\in\bar{C})\wedge(k\in\bar{B}\vee k\in A)$

$\wedge(k\in\bar{B}\vee k\in\bar{C})$

$\Rightarrow k\in A\wedge k\notin B\cap C$

$\Rightarrow k\in A-(B\cap C)$

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2) (15 points) Determine whether the following functions are one-to-one correspondence. If a function is one-to-one correspondence, determine its inverse.

a) $f : Z \to Z$ where $f(n) = \left\lceil \frac{n}{2} + 1 \right\rceil$.

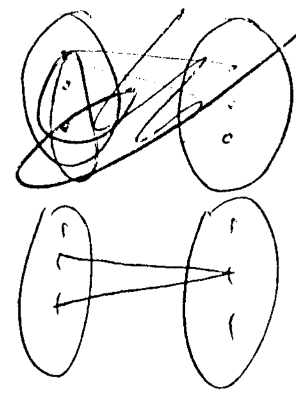
* $f(3) = \left\lceil \frac{3}{2} + 1 \right\rceil = \lceil 1.5 + 1 \rceil = \lceil 2.5 \rceil = 3$

* $f(4) = \left\lceil \frac{4}{2} + 1 \right\rceil = \lceil 2 + 1 \rceil = 3$

$\Rightarrow f$ is not a one to one funct: because for $\overset{3}{x} \neq \overset{4}{y}$

we have $f(\overset{3}{x}) = f(\overset{4}{y})$

$\Rightarrow$ no inverse.



b) $g : R \to R$ where $g(x) = x^3 + 1$.

Let ~~$f(n) = f(m)$~~ $g(n) = g(m) \Rightarrow n^3 + 1 = m^3 + 1$

$\Rightarrow n^3 = m^3$

$\Rightarrow n = m$

$\Rightarrow$ g is a one to one function

The inverse is: for $g(x) = y$

$\Rightarrow x^3 + 1 = y$.

$x^3 = y - 1 \Rightarrow x = \sqrt[3]{y-1}$

onto ?

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3) **(15 points)** Is it true that $(p \oplus q) \oplus r \leftrightarrow p \oplus (q \oplus r)$?

p	q	r	$p \oplus q$	$(p \oplus q) \oplus r$ (A)	$q \oplus r$	$p \oplus (q \oplus r)$ (B)	$A \leftrightarrow B$
T	T	T	F	T	F	T	T
T	T	F	F	F	T	F	T
T	F	T	T	F	T	F	T
T	F	F	T	T	F	T	T
F	T	T	T	F	F	F	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	F	F	F	T

True

4) **(10 points)** Prove or disprove that the sum of two irrational numbers is irrational.

Indirect prove: Let x and y be 2 rational numbers

let $x = \frac{a}{b}$ and $y = \frac{n}{m}$ 5

$\Rightarrow x + y = \frac{a}{b} + \frac{n}{m} = \frac{am + bn}{bm}$ is rational

$\Rightarrow$ sum of two irrational numbers is irrational,

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5) **(15 points)** Determine the validity of the following arguments.

If I study, then I will pass.
If I do not go to movie, then I will study.
I failed.

Therefore, I went to a movie.

p : I study
q : I pass
r : go to movie

$$\left.\begin{array}{l} p \longrightarrow q \\ \neg r \longrightarrow p \end{array}\right\} \neg r \rightarrow q \Leftrightarrow \neg q \rightarrow r$$

$\neg q$

$$\begin{array}{l} \neg q \\ \neg q \longrightarrow r \\ \hline \therefore r \end{array}$$

true

If I like mathematics, then I will study.
Either I don't study or I pass mathematics.
If I don't graduate, then I didn't pass mathematics.

If I graduate, then I studied.

p : Like mathematics
q : I study
r : pass mathematics
s : graduate

$$p \longrightarrow q$$

$$\left.\begin{array}{l} \neg q \vee r \Leftrightarrow q \longrightarrow r \\ \neg s \longrightarrow \neg r \Leftrightarrow r \rightarrow s \end{array}\right\} q \rightarrow s$$

$\Rightarrow p \rightarrow q$

$q \rightarrow s$

$\Rightarrow \boxed{p \rightarrow s}$ $\quad \therefore p \rightarrow s$

statement false

6) **(15 points)** Show, by mathematical induction, that

$$1^3 + 2^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

where n is a positive integer.

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for $n = 1 \Rightarrow \left(\frac{1\ (1+1)}{2}\right)^2 = 1 = 1^3$ is true for $n = 1$

suppose it is true for n; show that is true for $n+1$

$1^3 + 2^3 + \cdots + (n+1)^3 = \left[\frac{(n+1)(n+2)}{2}\right]^2$

$1^3 + 2^3 + \cdots + n^3 + (n+1)^3 = \left[\frac{n(n+1)}{2}\right]^2 + (n+1)^3 = \left[\frac{n^2+n}{2}\right]^2 + (n+1)^3$

$= \frac{n^4 + n^2 + 2n^3}{4} + n^3 + 3n^2 + 3n + 1 = \frac{n^4 + n^2 + 2n^3 + 4n^3 + 12n^2 + 12n + 4}{4}$

$= \frac{n^4 + 13n^2 + 6n^3 + 4}{4} = \left[\frac{(n+1)(n+2)}{2}\right]^2$

7) **(15 points)** Suppose that g is a function from A to B and f is a function from B to C. Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.

$g : A \to B$

$f : B \to C$

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Let $a_1, a_2 \in A \,/\, g(a_1) \neq g(a_2)$

$f(g(a_1)) = f(g(a_2)) \Rightarrow g(a_1) = g(a_2)$

$\Rightarrow f \circ g$ is also one-to-one.

Need to show: $(f \circ g)(a_1) = (f \circ g)(a_2) \Rightarrow a_1 = a_2$