Notre Dame University
Department of Mathematics and Statistics
MAT 339 (Numerical Analysis) Exam 2 Fall 2011
Duration: 50 minutes

1)(25%) The following table for $f(x)$ is given:

x	1	1.1	1.2	1.3	1.4
$f(x)$	1.548	1.67	1.81	1.97	2.15

Use all data values to approximate the value of c for which $f(c)=1.75$.

2)(25%) The backward-Euler formula is known to be a $O(h)$ approximation for $f'(x)$, and we have

$$f'(x)=\frac{f(x)-f(x-h)}{h}+\frac{h}{2}f''(x)-\frac{h^2}{6}f'''(x)+\frac{h^3}{24}f^{(4)}(x)-\ldots$$

Use Richardson's Extrapolation to derive a $O(h^3)$ approximation formula for $f'(x)$.

3)(25%) The well-know Simpson's rule with error term is

$$\int_a^b f(x)\,dx=\frac{h}{3}[f(x_0)+4f(x_1)+f(x_2)]-\frac{h^5}{90}f^{(4)}(\mu)\ldots\ldots(*)$$

where $x_0=a$, $x_1=a+h$, $x_2=b$, and $a<\mu<b$.

a) Use $(*)$ to derive the composite Simpson's rule with error term.

b) Use **(a)** to evaluate $J=\int_0^1 \frac{dx}{1+x^2}$ for $N=4$, and compare it with the exact value of *J*.

4)(25%) Recall that the degree of precision of a quadrature formula is the largest positive integer n such that the formula is exact for x^k, for each $k=0,1,2,\ldots,n$.

a) Derive a quadrature formula of the form

$$\int_{-2}^2 |x|f(x)\,dx\approx Af(-1)+Bf(0)+Cf(1)\ldots\ldots\ldots\ldots\ldots\ldots(**)$$

that is exact for polynomials of degree ≤ 2.

b) What is the degree of precision of (*)?