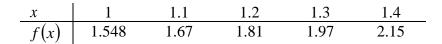
## Notre Dame University Department of Mathematics and Statistics MAT 339 (Numerical Analysis) Exam 2 Fall 2011 Duration: 50 minutes

**1)(25%)** The following table for f(x) is given:



Use all data values to approximate the value of c for which f(c) = 1.75.

**2)(25%)** The backward-Euler formula is known to be a O(h) approximation for f'(x), and we have

$$f'(x) = \frac{f(x) - f(x - h)}{h} + \frac{h}{2} f''(x) - \frac{h^2}{6} f'''(x) + \frac{h^3}{24} f^{(4)}(x) - \dots$$

Use Richardson's Extrapolation to derive a  $O(h^3)$  approximation formula for f'(x).

3)(25%) The well-know Simpson's rule with error term is

$$\int_{a}^{b} f(x) dx = \frac{h}{3} [f(x_{0}) + 4f(x_{1}) + f(x_{2})] - \frac{h^{5}}{90} f^{(4)}(\mu) \dots (*)$$

where  $x_0 = a$ ,  $x_1 = a + h$ ,  $x_2 = b$ , and  $a < \mu < b$ .

- a) Use (\*) to derive the composite Simpson's rule with error term.
- **b**) Use (a) to evaluate  $J = \int_0^1 \frac{dx}{1+x^2}$  for N = 4, and compare it with the exact value of J.

(25%) Recall that the degree of precision of a quadrature formula is the largest positive integer

*n* such that the formula is exact for  $x^k$ , for each k = 0, 1, 2, ..., n.

a) Derive a quadrature formula of the form

$$\int_{-2}^{2} |x| f(x) dx \approx Af(-1) + Bf(0) + Cf(1)....(**)$$

that is exact for polynomials of degree  $\leq 2$ .

**b**) What is the degree of precision of (\*)?