Math 201 - Exam 1 (Fall 08)

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- Please answer each question on the same sheet of paper on which it is written (after the line following the question). Any part of your answer written on the wrong page will not be graded.
- When finished leave your work on your desk for it to be collected by the proctors.
- There are 4 problems in total. Some questions have several parts to them. Make sure that you attempt them all.

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Name :

ID # :

Section Number :

Problem 1

(9 points each) Which of the following series converge and which diverge? For those which converge, do they converge conditionally or absolutely? When possible, find the sum of the series. You should justify your answers.

i- $$\sum_{n=0}^{\infty} \left\{ \frac{(-1)^n 3^n}{\pi^{n+1}} + \frac{2^{n+1}}{e^n} \right\}$$

ii- $$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^{1.0001}}$$

iii- $$\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n}$$

iv- $$\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

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Problem 2

(30 points) Write the Taylor series of $\ln(x)$ at $x = 1$. What is the radius of convergence of this series? Where does it converge absolutely and where conditionally? Prove that the Taylor series converges to $\ln(x)$ in its interval of convergence. At least how many terms should one take to estimate $\ln(2)$ with an error no greater than 10^{-6}?

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Problem 3

- (7 points) Prove that

$$\sum_{n=2}^{\infty} \frac{1}{n\,(\ln(n))^p}$$

converges when $p > 1$ and diverges when $0 \leq p \leq 1$.

- (7 points) Does the series

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n) \ln(\ln(n))}$$

converge or diverge?

- (7 points) Does the series

$$\sum_{n=2}^{\infty} \frac{1}{n \ln(n)\,(\ln(\ln(n)))^2}$$

converge or diverge?

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Problem 4

Consider the following power series

$$\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-1)^n}{4^n\,(2n+1)!}\;x^{4n+2}$$

(i) (7 points) What is the radius of convergence? Where does the series converge absolutely and where conditionally?

(ii) (6 points) Find the c_n's which solve the following equation

$$\left(1-\frac{1}{4}\Big(\sum_{n=0}^{\infty}\frac{(-1)^n}{4^n\,(2n+1)!}\;x^{4n+2}\Big)^2\right)^{\frac{1}{2}}=\sum_{n=0}^{\infty}c_n x^n$$

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