

A Brief Table of Integrals

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| <p>1. $\int u dv = uv - \int v du$</p> <p>3. $\int \cos u du = \sin u + C$</p> <p>5. $\int (ax + b)^n dx = \frac{(ax + b)^{n+1}}{a(n+1)} + C, \quad n \neq -1$</p> <p>7. $\int x(ax + b)^n dx = \frac{(ax + b)^{n+1}}{a^2} \left[\frac{ax + b}{n+2} - \frac{b}{n+1} \right] + C, \quad n \neq -1, -2$</p> <p>8. $\int x(ax + b)^{-1} dx = \frac{x}{a} - \frac{b}{a^2} \ln ax + b + C$</p> <p>10. $\int \frac{dx}{x(ax + b)} = \frac{1}{b} \ln \left \frac{x}{ax + b} \right + C$</p> <p>12. $\int \frac{\sqrt{ax + b}}{x} dx = 2\sqrt{\frac{ax + b}{x}} + b \int \frac{dx}{x\sqrt{ax + b}}$</p> <p>13. (a) $\int \frac{dx}{x\sqrt{ax - b}} = \frac{2}{\sqrt{b}} \tan^{-1} \sqrt{\frac{ax - b}{b}} + C$
 (b) $\int \frac{dx}{x\sqrt{ax + b}} = \frac{1}{\sqrt{b}} \ln \left \frac{\sqrt{ax + b} - \sqrt{b}}{\sqrt{ax + b} + \sqrt{b}} \right + C$</p> <p>14. $\int \frac{\sqrt{ax + b}}{x^2} dx = -\frac{\sqrt{ax + b}}{x} + \frac{a}{2} \int \frac{dx}{x\sqrt{ax + b}} + C$</p> <p>16. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$</p> <p>18. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left \frac{x+a}{x-a} \right + C$</p> <p>20. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1} \frac{x}{a} + C = \ln(x + \sqrt{a^2 + x^2}) + C$</p> <p>22. $\int x^2 \sqrt{a^2 + x^2} dx = \frac{x}{8} (a^2 + 2x^2) \sqrt{a^2 + x^2} - \frac{a^4}{8} \ln(x + \sqrt{a^2 + x^2}) + C$</p> <p>23. $\int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left \frac{a + \sqrt{a^2 + x^2}}{x} \right + C$</p> <p>25. $\int \frac{x^2}{\sqrt{a^2 + x^2}} dx = -\frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + \frac{x\sqrt{a^2 + x^2}}{2} + C$</p> <p>26. $\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a} \ln \left \frac{a + \sqrt{a^2 + x^2}}{x} \right + C$</p> | <p>2. $\int a^u du = \frac{a^u}{\ln a} + C, \quad a \neq 1, \quad a > 0$</p> <p>4. $\int \sin u du = -\cos u + C$</p> <p>6. $\int (ax + b)^{-1} dx = \frac{1}{a} \ln ax + b + C$</p> <p>9. $\int x(ax + b)^{-2} dx = \frac{1}{a^2} \left[\ln ax + b + \frac{b}{ax + b} \right] + C$</p> <p>11. $\int (\sqrt{ax + b})^n dx = \frac{2}{a} \frac{(\sqrt{ax + b})^{n+2}}{n+2} + C, \quad n \neq -2$</p> <p>15. $\int \frac{dx}{x^2 \sqrt{ax + b}} = -\frac{\sqrt{ax + b}}{bx} - \frac{a}{2b} \int \frac{dx}{x\sqrt{ax + b}} + C$</p> <p>17. $\int \frac{dx}{(a^2 + x^2)^2} = \frac{x}{2a^2(a^2 + x^2)} + \frac{1}{2a^3} \tan^{-1} \frac{x}{a} + C$</p> <p>19. $\int \frac{dx}{(a^2 - x^2)^2} = \frac{x}{2a^2(a^2 - x^2)} + \frac{1}{4a^3} \ln \left \frac{x+a}{x-a} \right + C$</p> <p>21. $\int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \ln(x + \sqrt{a^2 + x^2}) + C$</p> <p>24. $\int \frac{\sqrt{a^2 + x^2}}{x^2} dx = \ln(x + \sqrt{a^2 + x^2}) - \frac{\sqrt{a^2 + x^2}}{x} + C$</p> <p>27. $\int \frac{dx}{x^2 \sqrt{a^2 + x^2}} = -\frac{\sqrt{a^2 + x^2}}{a^2 x} + C$</p> |
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28. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$
29. $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$
30. $\int x^2 \sqrt{a^2 - x^2} dx = \frac{a^4}{8} \sin^{-1} \frac{x}{a} - \frac{1}{8} x \sqrt{a^2 - x^2} (a^2 - 2x^2) + C$
31. $\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$
32. $\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\sin^{-1} \frac{x}{a} - \frac{\sqrt{a^2 - x^2}}{x} + C$
33. $\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = \frac{a^2}{2} \sin^{-1} \frac{x}{a} - \frac{1}{2} x \sqrt{a^2 - x^2} + C$
34. $\int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$
35. $\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$
36. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1} \frac{x}{a} + C = \ln |x + \sqrt{x^2 - a^2}| + C$
37. $\int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + C$
38. $\int (\sqrt{x^2 - a^2})^n dx = \frac{x(\sqrt{x^2 - a^2})^n}{n+1} - \frac{na^2}{n+1} \int (\sqrt{x^2 - a^2})^{n-2} dx, \quad n \neq -1$
39. $\int \frac{dx}{(\sqrt{x^2 - a^2})^n} = \frac{x(\sqrt{x^2 - a^2})^{2-n}}{(2-n)a^2} - \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{x^2 - a^2})^{n-2}}, \quad n \neq 2$
40. $\int x(\sqrt{x^2 - a^2})^n dx = \frac{(\sqrt{x^2 - a^2})^{n+2}}{n+2} + C, \quad n \neq -2$
41. $\int x^2 \sqrt{x^2 - a^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{x^2 - a^2} - \frac{a^4}{8} \ln |x + \sqrt{x^2 - a^2}| + C$
42. $\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \sec^{-1} \left| \frac{x}{a} \right| + C$
43. $\int \frac{\sqrt{x^2 - a^2}}{x^2} dx = \ln |x + \sqrt{x^2 - a^2}| - \frac{\sqrt{x^2 - a^2}}{x} + C$
44. $\int \frac{x^2}{\sqrt{x^2 - a^2}} dx = \frac{a^2}{2} \ln |x + \sqrt{x^2 - a^2}| + \frac{x}{2} \sqrt{x^2 - a^2} + C$
45. $\int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C = \frac{1}{a} \cos^{-1} \left| \frac{a}{x} \right| + C$
46. $\int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \frac{\sqrt{x^2 - a^2}}{a^2 x} + C$
47. $\int \frac{dx}{\sqrt{2ax - x^2}} = \sin^{-1} \left(\frac{x-a}{a} \right) + C$
48. $\int \sqrt{2ax - x^2} dx = \frac{x-a}{2} \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + C$
49. $\int (\sqrt{2ax - x^2})^n dx = \frac{(x-a)(\sqrt{2ax - x^2})^n}{n+1} + \frac{na^2}{n+1} \int (\sqrt{2ax - x^2})^{n-2} dx$
50. $\int \frac{dx}{(\sqrt{2ax - x^2})^n} = \frac{(x-a)(\sqrt{2ax - x^2})^{2-n}}{(n-2)a^2} + \frac{n-3}{(n-2)a^2} \int \frac{dx}{(\sqrt{2ax - x^2})^{n-2}}$
51. $\int x \sqrt{2ax - x^2} dx = \frac{(x+a)(2x-3a)\sqrt{2ax - x^2}}{6} + \frac{a^3}{2} \sin^{-1} \left(\frac{x-a}{a} \right) + C$
52. $\int \frac{\sqrt{2ax - x^2}}{x} dx = \sqrt{2ax - x^2} + a \sin^{-1} \left(\frac{x-a}{a} \right) + C$
53. $\int \frac{\sqrt{2ax - x^2}}{x^2} dx = -2 \sqrt{\frac{2a-x}{x}} - \sin^{-1} \left(\frac{x-a}{a} \right) + C$
54. $\int \frac{x dx}{\sqrt{2ax - x^2}} = a \sin^{-1} \left(\frac{x-a}{a} \right) - \sqrt{2ax - x^2} + C$
55. $\int \frac{dx}{x \sqrt{2ax - x^2}} = -\frac{1}{a} \sqrt{\frac{2a-x}{x}} + C$
56. $\int \sin ax dx = -\frac{1}{a} \cos ax + C$
57. $\int \cos ax dx = \frac{1}{a} \sin ax + C$
58. $\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C$
59. $\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C$

$$60. \int \sin^n ax \, dx = -\frac{\sin^{n-1} ax \cos ax}{na} + \frac{n-1}{n} \int \sin^{n-2} ax \, dx$$

$$61. \int \cos^n ax \, dx = \frac{\cos^{n-1} ax \sin ax}{na} + \frac{n-1}{n} \int \cos^{n-2} ax \, dx$$

$$62. (a) \int \sin ax \cos bx \, dx = -\frac{\cos(a+b)x}{2(a+b)} - \frac{\cos(a-b)x}{2(a-b)} + C, \quad a^2 \neq b^2$$

$$(b) \int \sin ax \sin bx \, dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$(c) \int \cos ax \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C, \quad a^2 \neq b^2$$

$$63. \int \sin ax \cos ax \, dx = -\frac{\cos 2ax}{4a} + C$$

$$64. \int \sin^n ax \cos ax \, dx = \frac{\sin^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$$

$$65. \int \frac{\cos ax}{\sin ax} \, dx = \frac{1}{a} \ln |\sin ax| + C$$

$$66. \int \cos^n ax \sin ax \, dx = -\frac{\cos^{n+1} ax}{(n+1)a} + C, \quad n \neq -1$$

$$67. \int \frac{\sin ax}{\cos ax} \, dx = -\frac{1}{a} \ln |\cos ax| + C$$

$$68. \int \sin^n ax \cos^m ax \, dx = -\frac{\sin^{n-1} ax \cos^{m+1} ax}{a(m+n)} + \frac{n-1}{m+n} \int \sin^{n-2} ax \cos^m ax \, dx, \quad n \neq -m \quad (\text{reduces } \sin^n ax)$$

$$69. \int \sin^n ax \cos^m ax \, dx = \frac{\sin^{n-1} ax \cos^{m-1} ax}{a(m+n)} + \frac{m-1}{m+n} \int \sin^n ax \cos^{m-2} ax \, dx, \quad m \neq -n \quad (\text{reduces } \cos^m ax)$$

$$70. \int \frac{dx}{b+c \sin ax} = \frac{-2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[\sqrt{\frac{b-c}{b+c}} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) \right] + C, \quad b^2 > c^2$$

$$71. \int \frac{dx}{b+c \sin ax} = \frac{-1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \sin ax + \sqrt{c^2-b^2} \cos ax}{b+c \sin ax} \right| + C, \quad b^2 < c^2$$

$$72. \int \frac{dx}{1+\sin ax} = -\frac{1}{a} \tan \left(\frac{\pi}{4} - \frac{ax}{2} \right) + C$$

$$73. \int \frac{dx}{1-\sin ax} = \frac{1}{a} \tan \left(\frac{\pi}{4} + \frac{ax}{2} \right) + C$$

$$74. \int \frac{dx}{b+c \cos ax} = \frac{2}{a\sqrt{b^2-c^2}} \tan^{-1} \left[\sqrt{\frac{b-c}{b+c}} \tan \frac{ax}{2} \right] + C, \quad b^2 > c^2$$

$$75. \int \frac{dx}{b+c \cos ax} = \frac{1}{a\sqrt{c^2-b^2}} \ln \left| \frac{c+b \cos ax + \sqrt{c^2-b^2} \sin ax}{b+c \cos ax} \right| + C, \quad b^2 < c^2$$

$$76. \int \frac{dx}{1+\cos ax} = \frac{1}{a} \tan \frac{ax}{2} + C$$

$$77. \int \frac{dx}{1-\cos ax} = -\frac{1}{a} \cot \frac{ax}{2} + C$$

$$78. \int x \sin ax \, dx = \frac{1}{a^2} \sin ax - \frac{x}{a} \cos ax + C$$

$$79. \int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax + C$$

$$80. \int x^n \sin ax \, dx = -\frac{x^n}{a} \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$

$$81. \int x^n \cos ax \, dx = \frac{x^n}{a} \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$$

$$82. \int \tan ax \, dx = \frac{1}{a} \ln |\sec ax| + C$$

$$83. \int \cot ax \, dx = \frac{1}{a} \ln |\sin ax| + C$$

$$84. \int \tan^2 ax \, dx = \frac{1}{a} \tan ax - x + C$$

$$85. \int \cot^2 ax \, dx = -\frac{1}{a} \cot ax - x + C$$

$$86. \int \tan^n ax \, dx = \frac{\tan^{n-1} ax}{a(n-1)} - \int \tan^{n-2} ax \, dx, \quad n \neq 1$$

$$87. \int \cot^n ax \, dx = -\frac{\cot^{n-1} ax}{a(n-1)} - \int \cot^{n-2} ax \, dx, \quad n \neq 1$$

$$88. \int \sec ax \, dx = \frac{1}{a} \ln |\sec ax + \tan ax| + C$$

$$89. \int \csc ax \, dx = -\frac{1}{a} \ln |\csc ax + \cot ax| + C$$

$$90. \int \sec^2 ax \, dx = \frac{1}{a} \tan ax + C$$

$$91. \int \csc^2 ax \, dx = -\frac{1}{a} \cot ax + C$$

$$92. \int \sec^n ax \, dx = \frac{\sec^{n-2} ax \tan ax}{a(n-1)} + \frac{n-2}{n-1} \int \sec^{n-2} ax \, dx, \quad n \neq 1$$

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93. $\int \csc^n ax \, dx = -\frac{\csc^{n-2} ax \cot ax}{a(n-1)} + \frac{n-2}{n-1} \int \csc^{n-2} ax \, dx, \quad n \neq 1$
94. $\int \sec^n ax \tan ax \, dx = \frac{\sec^n ax}{na} + C, \quad n \neq 0$
95. $\int \csc^n ax \cot ax \, dx = -\frac{\csc^n ax}{na} + C, \quad n \neq 0$
96. $\int \sin^{-1} ax \, dx = x \sin^{-1} ax + \frac{1}{a} \sqrt{1-a^2x^2} + C$
97. $\int \cos^{-1} ax \, dx = x \cos^{-1} ax - \frac{1}{a} \sqrt{1-a^2x^2} + C$
98. $\int \tan^{-1} ax \, dx = x \tan^{-1} ax - \frac{1}{2a} \ln(1+a^2x^2) + C$
99. $\int x^n \sin^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \sin^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{\sqrt{1-a^2x^2}}, \quad n \neq -1$
100. $\int x^n \cos^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \cos^{-1} ax + \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{\sqrt{1-a^2x^2}}, \quad n \neq -1$
101. $\int x^n \tan^{-1} ax \, dx = \frac{x^{n+1}}{n+1} \tan^{-1} ax - \frac{a}{n+1} \int \frac{x^{n+1} \, dx}{\sqrt{1+a^2x^2}}, \quad n \neq -1$
102. $\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C$
103. $\int b^{ax} \, dx = \frac{1}{a} \frac{b^{ax}}{\ln b} + C, \quad b > 0, \quad b \neq 1$
104. $\int x e^{ax} \, dx = \frac{e^{ax}}{a^2} (ax - 1) + C$
105. $\int x^n e^{ax} \, dx = \frac{1}{a} x^n e^{ax} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$
106. $\int x^n b^{ax} \, dx = \frac{x^n b^{ax}}{a \ln b} - \frac{n}{a \ln b} \int x^{n-1} b^{ax} \, dx, \quad b > 0, \quad b \neq 1$
107. $\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$
108. $\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$
109. $\int \ln ax \, dx = x \ln ax - x + C$
110. $\int x^n (\ln ax)^m \, dx = \frac{x^{n+1} (\ln ax)^m}{n+1} - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} \, dx, \quad n \neq -1$
111. $\int x^{-1} (\ln ax)^m \, dx = \frac{(\ln ax)^{m+1}}{m+1} + C, \quad m \neq -1$
112. $\int \frac{dx}{x \ln ax} = \ln |\ln ax| + C$
113. $\int \sinh ax \, dx = \frac{1}{a} \cosh ax + C$
114. $\int \cosh ax \, dx = \frac{1}{a} \sinh ax + C$
115. $\int \sinh^2 ax \, dx = \frac{\sinh 2ax}{4a} - \frac{x}{2} + C$
116. $\int \cosh^2 ax \, dx = \frac{\sinh 2ax}{4a} + \frac{x}{2} + C$
117. $\int \sinh^n ax \, dx = \frac{\sinh^{n-1} ax \cosh ax}{na} - \frac{n-1}{n} \int \sinh^{n-2} ax \, dx, \quad n \neq 0$
118. $\int \cosh^n ax \, dx = \frac{\cosh^{n-1} ax \sinh ax}{na} + \frac{n-1}{n} \int \cosh^{n-2} ax \, dx, \quad n \neq 0$
119. $\int x \sinh ax \, dx = \frac{x}{a} \cosh ax - \frac{1}{a^2} \sinh ax + C$
120. $\int x \cosh ax \, dx = \frac{x}{a} \sinh ax - \frac{1}{a^2} \cosh ax + C$
121. $\int x^n \sinh ax \, dx = \frac{x^n}{a} \cosh ax - \frac{n}{a} \int x^{n-1} \cosh ax \, dx$
122. $\int x^n \cosh ax \, dx = \frac{x^n}{a} \sinh ax - \frac{n}{a} \int x^{n-1} \sinh ax \, dx$
123. $\int \tanh ax \, dx = \frac{1}{a} \ln |\cosh ax| + C$
124. $\int \coth ax \, dx = \frac{1}{a} \ln |\sinh ax| + C$
125. $\int \tanh^2 ax \, dx = x - \frac{1}{a} \tanh ax + C$
126. $\int \coth^2 ax \, dx = x - \frac{1}{a} \coth ax + C$
127. $\int \tanh^n ax \, dx = -\frac{\tanh^{n-1} ax}{(n-1)a} + \int \tanh^{n-2} ax \, dx, \quad n \neq 1$
128. $\int \coth^n ax \, dx = -\frac{\coth^{n-1} ax}{(n-1)a} + \int \coth^{n-2} ax \, dx, \quad n \neq 1$
129. $\int \operatorname{sech} ax \, dx = \frac{1}{a} \sin^{-1} (\tanh ax) + C$
130. $\int \operatorname{csch} ax \, dx = \frac{1}{a} \ln \left| \tanh \frac{ax}{2} \right| + C$