N.D.U **MAT 112** **Second Test** **Fall 2001 – 2002** **December 12, 2001**

Name: **ID #:** _

Section: A **Time: 60 minutes**

Directions
1. Write neatly and clearly.
2. Do not use pencils unless for graphing.
3. Show all work.
4. No calculators are allowed.
5. Turn off your mobile phones.

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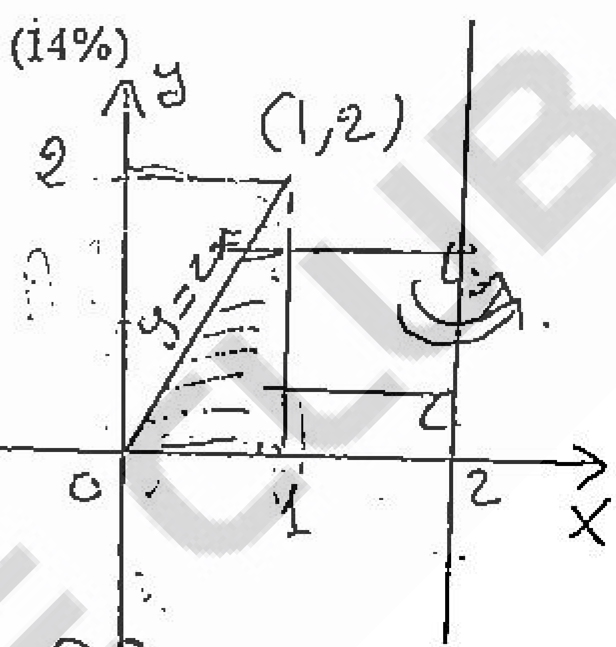
I. Find the volume of the solid generated by revolving the triangular region bounded by the lines $y = 2x$, $y = 0$ and $x = 1$, about the line $x = 2$. Use the Washer Method only. (14%)

$x = \frac{y}{2}$

~~Inner~~ Outer radius: ~~$2 - y = 2 - 2x$~~

$R(y) = 2 - x = 2 - \frac{y}{2}$ (2)

~~Outer~~ Inner radius: $\cancel{R(y)} \cancel{x}$ $r(y) = 2 - 1 = 1$.



(5)

$V = \pi \int_0^2 (R(y))^2 - (r(y))^2 dy$

$= \pi \int_0^2 4 - (2 - \frac{y}{2})^2 dy$

$= \pi \int_0^2 [4 - (4 + \frac{y^2}{4} - 2y)] dy$ X

* $V = \pi \int_0^2 [(2 - \frac{y}{2})^2 - 1^2]$

$= \pi \int_0^2 (4 - 4 - \frac{y^2}{4} + 2y) dy$

$= \pi \int_0^2 (2y - \frac{y^2}{4}) dy = \pi [y^2 - \frac{y^3}{12}]_0^2$ X $= \pi \int_0^2 [\frac{y^2}{4} - 2y + ...$

$= (4 - \frac{8}{12})\pi = (4 - \frac{2}{3})\pi = (\frac{12 - 2}{3})\pi = \boxed{\frac{10\pi}{3} u^3}$ X $= \frac{8\pi}{3} u^3$

II. Use the <u>Shell method</u> to find the volume of the solid generated by revolving the region bounded by the curve $y = 2 - x^2$, $y = x^2$ and $x = 0$ about ~~the x-axis~~. y-axis

(14 %)

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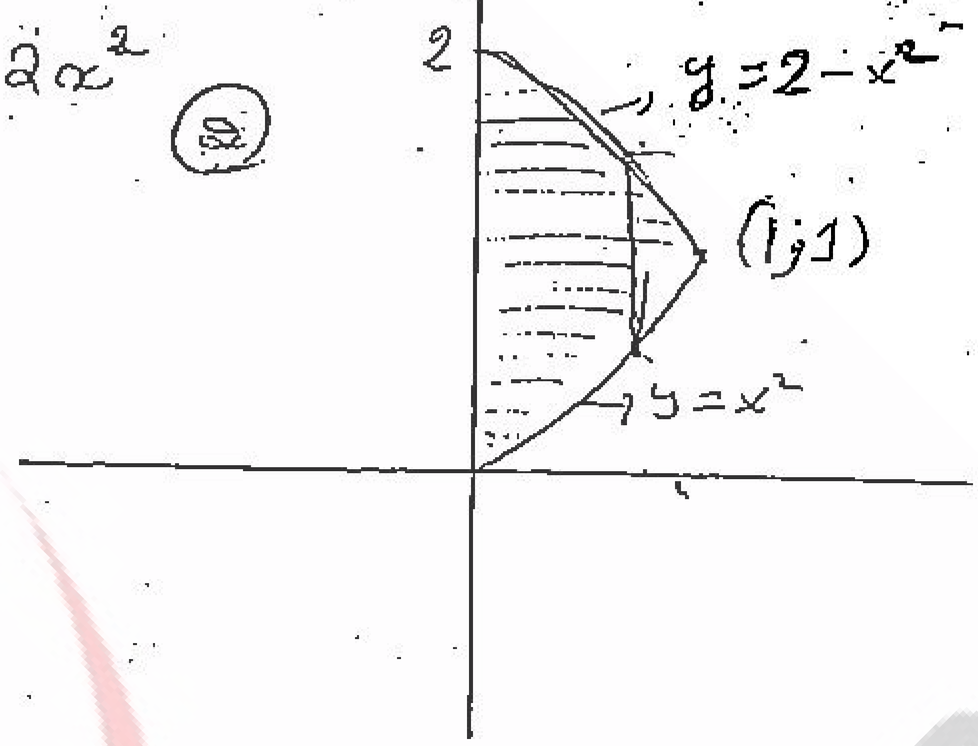
Shell radius : x 2

Shell high : $2 - x^2 - x^2 = 2 - 2x^2$ 2



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$$V = 2\pi \int_0^1 x\,(2 - 2x^2)\,dx$$

$$= 2\pi \int_0^1 (2x - 2x^3)\,dx$$

4

$$= 2\pi \left[x^2 - \frac{2x^4}{4} \right]_0^1 = 2\pi \left[x^2 - \frac{x^4}{2} \right]_0^1 = 2\pi\left(1 - \frac{1}{2}\right) = \frac{2\pi}{2} = \pi \text{ u}$$

$$\cancel{= 2\pi\left(4 - \frac{16}{2}\right) = 2\pi(4-8) = \boxed{-8\pi x^3}}$$

III. Given a thin plate covering the region bounded by the parabola $y = x^2$ and above by the line $y = x$; the plate's density at (x, y) is $\delta(x) = 12x$.

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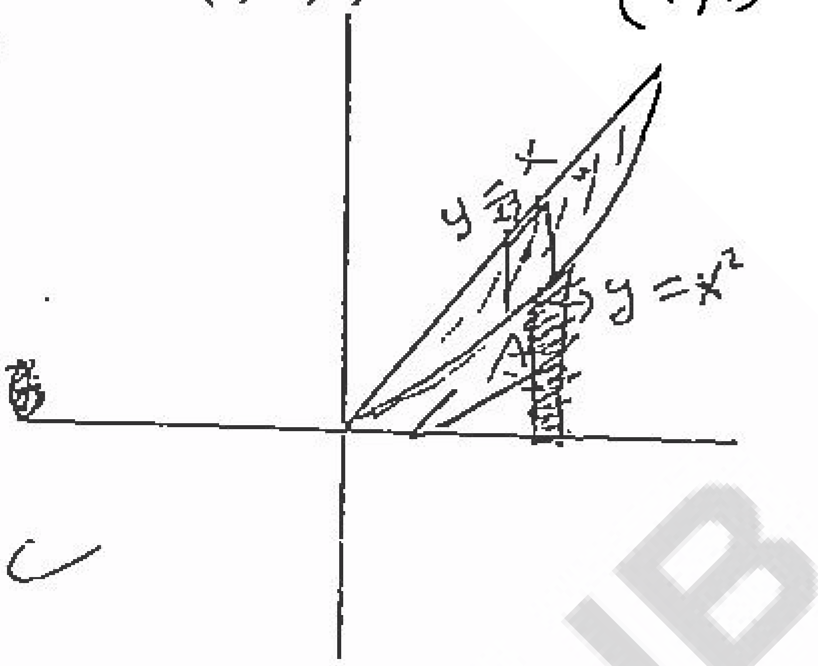
a) Find the total mass of this region.
b) Find the moment of this region about the x-axis.
c) Deduce the ordinate of the center of mass of this region.

(22 %)
(6, 13, 3)

$dA = (x - x^2)\,dx$ 3

$y = y \qquad x_1 = 0$
$x^2 = x \qquad x_2 = 1$

(1,1)



a) 6 $M = \int_0^1 \delta(x)\,dA = \int_0^1 12x(x - x^2)\,dx$ 3

$= 12\int_0^1 (x^2 - x^3)\,dx = 12\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$

$= 12\left(\frac{1}{3} - \frac{1}{4}\right) = 12\left(\frac{4-3}{12}\right) = \boxed{1}$

b) $M_x = ?$ $\tilde{y} = \frac{x^2 + x}{2}$ 2

13 5

$\cancel{M_x = \int_0^1 \tilde{y}\,\delta(x)\,dA = \int_0^1 \frac{x^2+x}{2} \cdot 12x \cdot (x - x^2)\,dx}$

$\cancel{= 6\int_0^1 [(x^4 + x^2)x]\,dx = 6\int_0^1 (x^5 + x^3)\,dx = 6\left[\frac{x^6}{6} + \frac{x^4}{4}\right]_0^1}$

$\cancel{= 6\left(\frac{1}{6} + \frac{1}{4}\right) = 6\left(\frac{4+6}{24}\right) = \frac{60}{24} = \frac{5}{2}}$

$\cancel{c)\ \bar{y} = \frac{M_x}{M} = \frac{5/2}{1} = \frac{5}{2} \times \frac{1}{7} = \frac{5}{14}}$

$M_x = 6\int_0^1 x(x^2 - x^4)\,dx = 6\int_0^1 (x^3 - x^5)\,dx = \frac{6}{12} = \frac{1}{2}$.

b) $M_x = \int_0^1 \delta_x\,\tilde{y}\,dA = \int_0^1 \frac{x^2+x}{2} \cdot 12x(x - x^2)\,dx$

$= 6\int_0^1 (x^3 - x^5)\,dx$

$= 6\left[\frac{x^4}{4} - \frac{x^6}{6}\right]$

$= 6\left(\frac{1}{4} - \frac{1}{6}\right)$ 6

$= \frac{1}{2}$

3 c)

$\bar{y} = \frac{\frac{1}{2}}{1} = \frac{1}{2}$ 3

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IV. Given the curve $y=\frac{1}{3}\left(x^2+2\right)^{3/2}$ .... (C).

Find the length of the curve between $x=0$ and $x=3$. (14%)

4

$$\frac{dy}{dx}=\frac{1}{3}\cdot\frac{3}{2}(2x)\left(x^2+2\right)^{1/2}=x\sqrt{x^2+2}$$

$$\left(\frac{dy}{dx}\right)^2=x^2\left(x^2+2\right)=x^4+2x^2$$

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$$L=\int_0^3\sqrt{1+x^4+2x^2}\,dx=\int_0^3\sqrt{\left(x^2+1\right)^2}\,dx=\int_0^3\left(x^2+1\right)dx$$

4

$$=\left[\frac{x^3}{3}+x\right]_0^3=\frac{27}{3}+3=9+3=\cancel{8}\,\cancel{6}\,12.$$

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V. Revolve $y = \sqrt{2x+1}$; $0 \le x \le 3$ around the x-axis.
Find the area of the generated surface. (12%)

$$S = \int_0^3 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \quad 4$$

$$y = \sqrt{2x+1} = (2x+1)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(2)(2x+1)^{-1/2}$$

$$= \frac{1}{\sqrt{2x+1}} \quad 2$$

$$S = 2\pi \int_0^3 \left(\sqrt{2x+1}\sqrt{1 + \frac{1}{2x+1}}\right) dx$$

$$= 2\pi \int_0^3 \left(\sqrt{2x+1}\sqrt{\frac{2x+2}{2x+1}}\right) dx$$

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{2x+1}$$

3

$$= 2\pi \int_0^3 \sqrt{(2x+2)}\, dx = \cancel{\pi\left[x^2 + 2x\right]_0^3 = \pi[9+6] = 15\pi}$$

Let $u = 2x + 2 \Rightarrow du = 2dx$.

$\underline{x = 0} \Rightarrow u = 2$; $\underline{x = 3} \Rightarrow u = 8$.

So $S = \dfrac{2\pi}{2} \displaystyle\int_2^8 u^{1/2}\, du = \dfrac{28\pi\sqrt{2}}{3}$

VI. The given region R revolved about the x-axis, generates half a sphere of volume $= \frac{2\pi a^3}{3}$. Use Pappus Theorem to find the centroid of the region R knowing that R has $y = x$ as an axis of symmetry.

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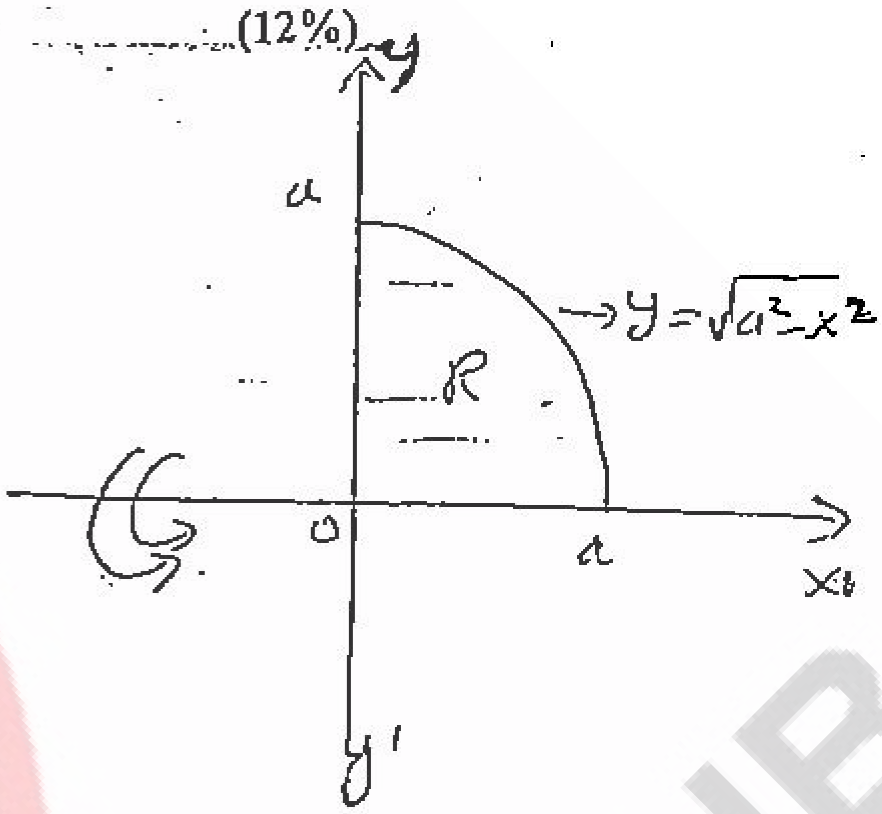
(12%)



$V = \frac{2\pi a^3}{3}$

$V = 2\pi A \rho$ 4

$A = \frac{\pi R^2}{4} = \frac{\pi a^2}{4}$ 3

$\frac{2\pi a^3}{3} = \frac{\pi a^2}{2}\rho$ (2π)

$\rho = \frac{4\pi a^3}{3\pi a^2 (2\pi)} = \frac{4}{3}\frac{a}{\pi}$ 1

$\bar{x} = \bar{y} = \frac{4a}{3\pi}$

so $(\bar{x}, \bar{y}) = \left(\frac{4a}{3\pi}; \frac{4a}{3\pi}\right)$ 3

VII. If a force of 20 lb is required to hold a Spring 1 foot beyond its unstressed length, how much work does it take to stretch the spring this far? One additional foot?

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(12%)

$F = 20\ lb \longrightarrow 1\ ft$

$F = 20x \longleftarrow x\ ft$

$W = \int_0^1 F(x)\,dx = \int_0^1 20x\,dx = \left[10x^2\right]_0^1 = 10\ ft.lb.$

4 4

For an additional foot:

$W_1 = \int_1^2 F(x)\,dx = \int_1^2 20x\,dx = \left[10x^2\right]_1^2 = 40 - 10 = 30\ ft.lb.$

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