50 Calculus II

N.D.U

MAT 112
First Test

Name:

ID #:

Section: A

Time: 60 minutes

96 ~~98~~

Directions

1. Write neatly and clearly.
2. Do not use pencils unless for graphing.
3. Show all work.
4. No calculators are allowed.
5. Turn off your mobile phones.

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I. Evaluate $\int 4\sec\left(\frac{x}{2}\right).\tan\left(\frac{x}{2}\right)dx$. (10 points)

$= 4\int \sec\left(\frac{x}{2}\right)\cdot\tan\left(\frac{x}{2}\right)dx = 4 \times \frac{1}{\frac{1}{2}}\left(\sec\frac{x}{2}\right) = \boxed{8\sec\frac{x}{2} + C}$

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II. Evaluate $\int x^{1/3}\sin\left(x^{4/3}-8\right)dx$. (8 points)

let $u = x^{4/3} - 8$ $\quad du = \frac{4}{3}x^{\frac{4}{3}-1}dx = \frac{4}{3}x^{1/3}dx \Rightarrow \frac{3}{4}du = x^{\frac{1}{3}}dx$

so $\int x^{\frac{1}{3}}\sin(x^{\frac{4}{3}} - 8)dx = \frac{3}{4}\int du \sin u = \boxed{\frac{3}{4}\cos(x^{\frac{4}{3}} - 8) + C}$

III. Solve the following initial value problem:

$$\frac{d^2y}{dx^2} = 2 - 6x; \qquad y'(0) = 4 \text{ and } y(0) = 1$$ **(12 points)**

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$$\int \frac{d^2y\,dx}{dx^2} = \int (2 - 6x)\,dx = \frac{dy}{dx} = 2x - \frac{6x^2}{2} + c = 2x - 3x^2 + c$$

$y'(0) = 4$ So $4 = 0 + 0 + c$ $\boxed{c = 4}$ then $\boxed{\frac{dy}{dx} = 2x - 3x^2 + 4}$

$$\int \frac{dy}{dx} \cdot dx = \int (2x - 3x^2 + 4)\,dx = y = \frac{2x^2}{2} - \frac{3x^3}{3} + 4x + c = x^2 - x^3 + 4x + c$$

$= x^2 - x^3 + 4x + c$

$y(0) = 1$ So $1 = 0 - 0 + 0 + c$ $\boxed{c = 1}$ ✓

then $\boxed{y = x^2 - x^3 + 4x + 1}$ ✓

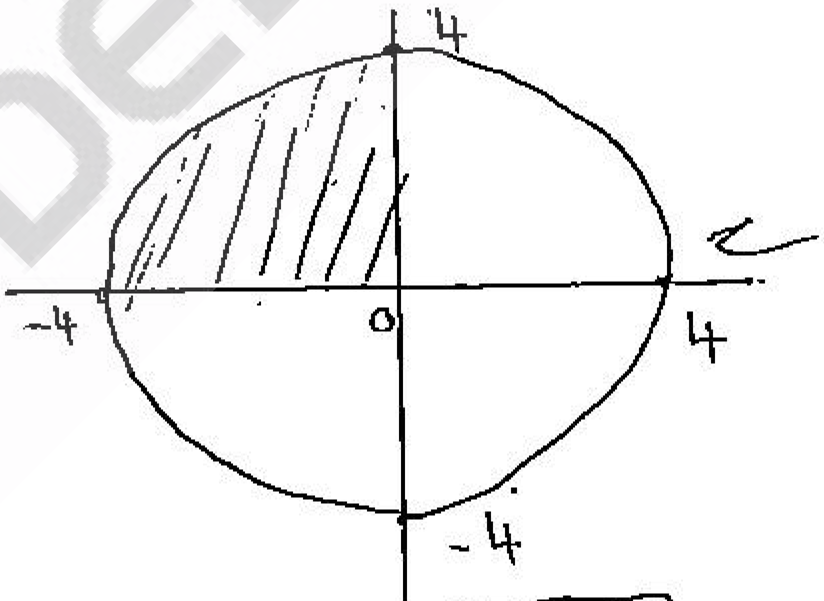
IV. Graph the integrand and use area to evaluate the integral: $\int_{-4}^{0} \sqrt{16 - x^2}\,dx$

(10 points)

$y = \sqrt{16 - x^2}$ $\quad y^2 = 16 - x^2 \Rightarrow x^2 + y^2 = 16$

It is the equation of the circle wich center is $I(0,0)$ and radius 4.

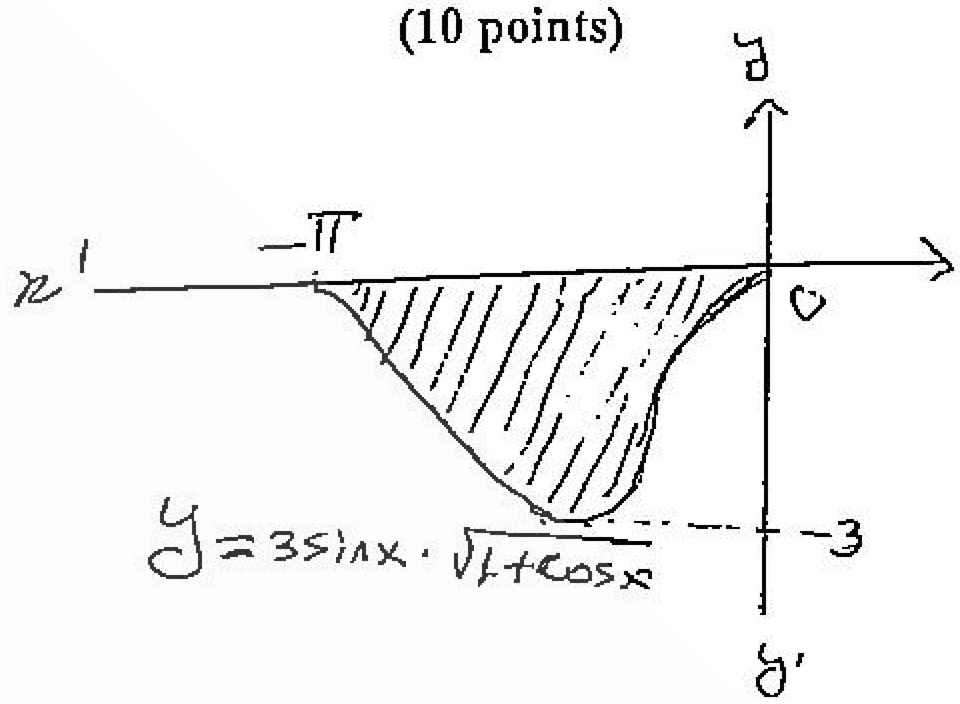
The required area is between -4 and 0 $[-4, 0]$ and wee have $y = \sqrt{16 - x^2}$ then;



$$\int_{-4}^{0} \sqrt{16 - x^2}\,dx = \frac{1}{4}\pi R^2 = \frac{1}{4}\pi\, 16 = \boxed{4\pi\ u^2}$$ = Area of $\frac{1}{4}$ of the cir

V. Find the area of the following shaded region. (10 points)

$f(x) = y = 3\sin x \cdot \sqrt{1+\cos x}$



Area of this region is:

$$-\int_{-\pi}^{0} f(x)\,dx = -\int_{-\pi}^{0} \left(3\sin x \cdot \sqrt{1+\cos x}\right)dx$$

$$= -3\int_{-\pi}^{0} \left(\sin x \cdot \sqrt{1+\cos x}\right)dx$$

let $u = 1+\cos x$ $\quad du = -\sin x\,dx$

$$= 3\int_{-\pi}^{0} u^{\frac{1}{2}}\,du = -3\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_{-\pi}^{0} = -3 \times \frac{2}{3}\left[u^{\frac{3}{2}}\right]_{-\pi}^{0} = -2\left[(1+\cos x)\sqrt{1+\cos x}\right.$$

$$= -2\left[0 + (2)\sqrt{2}\right] = \boxed{4\sqrt{2}}\,u^2$$

$$\left[(1+\cos(0))\sqrt{1+\cos(0)} - (1+\cos(-\pi))\sqrt{1+\cos(-\pi)}\right] = \left[2\sqrt{2} - (1-1)\sqrt{0}\right] = 2\sqrt{2}$$

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VI. Use the inequality $\sec(x) \geq 1 + \frac{x^2}{2}$ which is true in $[0,1]$ to find a lower bound for the value of $\int_0^1 \sec x\,dx$. (8 points)

$\sec(x) \geq 1 + \frac{x^2}{2}$ in $[0,1]$ so

$$\int_0^1 \sec x\,dx \geq \int_0^1 \left(1+\frac{x^2}{2}\right)dx = \left[x + \frac{1}{2}\frac{x^3}{3}\right]_0^1 = \left[x + \frac{x^3}{6}\right]_0^1$$

$$= 1 + \frac{1}{6} = \boxed{\frac{7}{6}}$$ is the lower bound for $\int_0^1 \sec x\,dx$

VII. Given $y = \int_0^{x^2} \cos(\sqrt{t})\,dt$. Find $\dfrac{dy}{dx}$. (10 points)

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$$\frac{dy}{dx} = \frac{d}{dx}\int_0^{x^2} \cos\sqrt{t}\,dt \qquad \text{let } u = x^2 \quad du = 2x\,dx$$

$$= \frac{d}{dx}\cdot\frac{du}{dx} = 2x \cdot \cos\sqrt{x^2} = \boxed{2x\cos|x| = \frac{dy}{dx}} =$$

$$= 2x\cos(x),$$

since $\cos(-\alpha) = \cos(\alpha)$.

VIII. Find the area of the following shaded region: (14 points)

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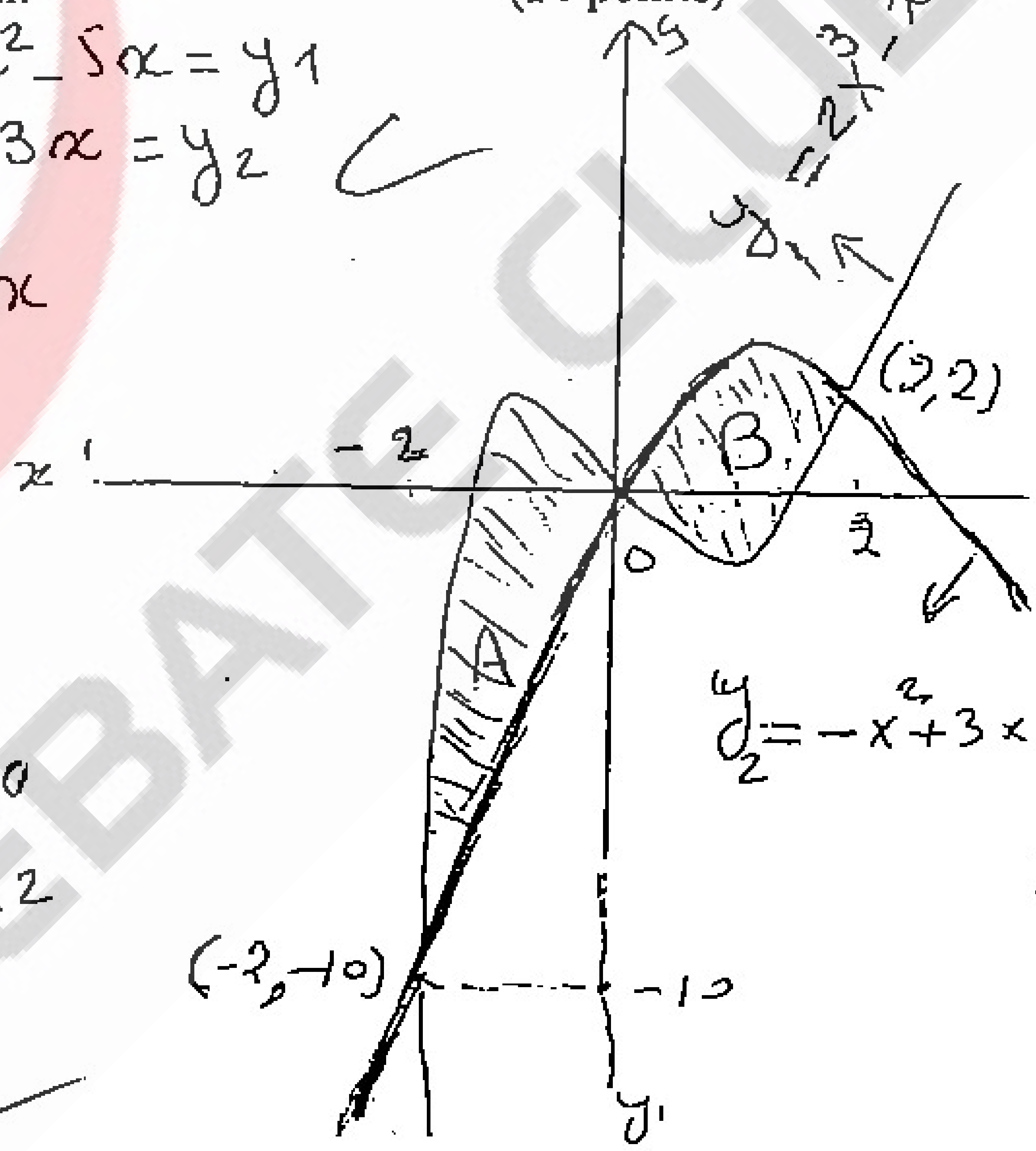
In A: ~~upper~~ curve $= f(x) = 2x^3 - x^2 - 5x = y_1$

Lower " $= g(x) = -x^2 + 3x = y_2$

$$f(x) - g(x) = 2x^3 - x^2 - 5x + x^2 - 3x$$
$$= 2x^3 - 8x$$

$$\text{Area of } A = \int_{-2}^{0} f(x) - g(x)\,dx$$
$$= \int_{-2}^{0} (2x^3 - 8x)\,dx$$
$$= \left[\frac{2x^4}{4} - 8\frac{x^2}{2}\right]_{-2}^{0} = \left[\frac{x^4}{2} - 4x^2\right]_{-2}^{0}$$
$$0 - \left(\frac{16}{2} - 16\right) = -(8 - 16)$$
$$= 8$$



In B, Lower curve $2x^3 - x^2 - 5x = g(x) = y_1$

Upper curve $-x^2 + 3x = f(x) = y_2$

$$f(x) - g(x) = -x^2 + 3x - 2x^3 + x^2 + 5x = -2x^3 + 8x$$

$$\text{Area of } B = \int_0^2 (-2x^3 + 8x)\,dx = \left[-\frac{2x^4}{4} + \frac{8x^2}{2}\right]_0^2 = \left[-\frac{x^4}{2} + 4x^2\right]_0^2$$

$$= -\frac{16}{2} + 16 - 0 = 8$$

Total Area = Area of A + Area of B = 8 + 8 = $\boxed{16\ u^2}$

IX. Consider a solid that lies between planes perpendicular to the x-axis at $x = 0$ and $x = 4$. The cross sections of the solid are perpendicular to the x-axis and are between these planes, and run from the parabola $y = -\sqrt{x}$ to the parabola $y = \sqrt{x}$. These cross sections are circular disks with diameters in the xy-plane. Find a formula for the area $A(x)$ of the cross sections, then use it to get the volume V of the solid.

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(18 points)

$AB = 2R = 2\sqrt{x}$ = diameter

$R = \frac{2\sqrt{x}}{2} = \sqrt{x}$

$A(x)$ = Area of the circle

$= \pi R^2 = \pi x \; u^2$

Volume of the solid

$V = \int_0^4 A(x)\,dx = \int_0^4 (\pi x)\,dx$

$= \pi \int_0^4 x\,dx = \pi\left[\frac{x^2}{2}\right]_0^4$

$= \pi\left(\frac{16}{2}\right) = \boxed{8\pi \, u^3}$