

Tuesday April 9, 2002

1) (18 points) Evaluate the following integrals.

a) $\int \sin \frac{x}{2} \cos^3 \frac{x}{2} dx$

b) $\int_0^2 \sqrt{4-x^2} dx$

c) $\int \frac{(1-\sqrt{x})^4}{\sqrt{x}} dx$

2) (10 points) Find the mean value of the function $f(x) = |x^2 - 1|$ over the interval $[0, 2]$.

3) (10 points) If $\int_0^x f(t) dt = \sin x + \tan x$. Find $f\left(\frac{\pi}{4}\right)$.

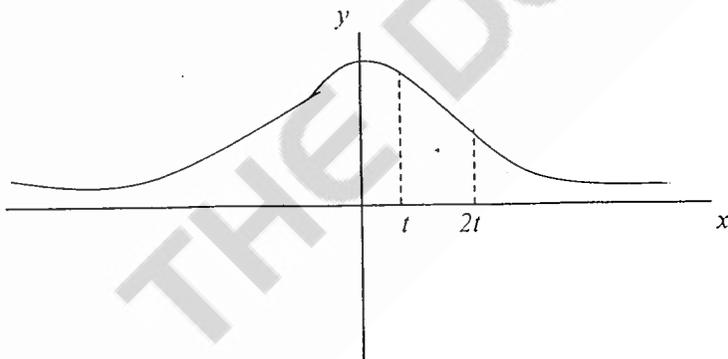
4) (16 points) Find the area of the region between the graph of $f(x) = x^2 - 4$ and the x -axis from $x = 0$ to $x = 3$.

5) (26 points) Find the volume of the solid generated by revolving the region enclosed by the curve $y = \sqrt{2x}$, the line $x = 8$ and the line $y = 2$, around

a) The line $y = 2$.

b) The y -axis.

6) (20 points) Below is the graph of the function $f(x) = \frac{1}{x^2 + 1}$.



a) If t is a real positive variable and $y(t)$ is defined as $y(t) = \int_t^{2t} f(x) dx$, find $\frac{dy}{dt}$.

b) $y(t)$ (Defined in part (a)) represents the area of the region between the graph of $f(x)$ and x -axis from t to $2t$. Find t that maximizes this area.