Quiz 1

PHYS 237: Introduction to Plasma Physics

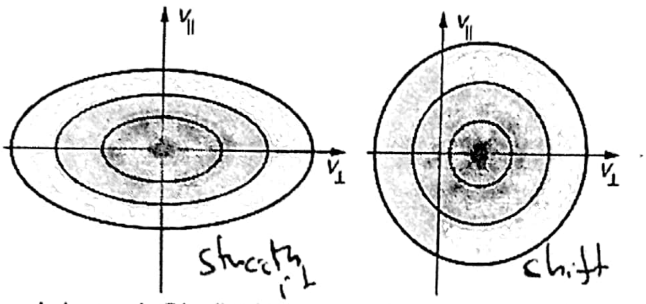
Spring 2018-2019

1- (10) Consider a plasma with a Maxwellian distribution.

a. (5) Show that the average velocity of the below function is v_0

$$f(\mathrm{v}) = n\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{m(\mathrm{v}-\mathrm{v}_0)^2}{2k_B T}\right)$$

b. (5) Indicate for the distributions in the image below what would be the form of the distribution functions



Anisotropic Distribution   Drifting Maxwellian

Give Maxwellian EQ For both graphs

2- (20) Consider a charge q with an arbitrary velocity $\mathbf{v}_0$ immersed in a uniform magnetic field **B**.

a. (5) Obtain the equation of motion of this charge in the three directions.

b. (5) Describe the motion parallel to B

c. (5) Show that $v_x = A\sin(\omega t)$ is a solution of the equation of motion in the direction perpendicular to **B**. Determine A and ω.

d. (5) Demonstrate the motion of the *guiding center* is uniform.

3- (30) Consider a neutral plasma with ions and electrons having respectively temperatures T_i and T_e. At the origin of this plasma, we add an addition positive charge +Q.

a. (5) Write down the Boltzmann relation for the electrons and ions and comment.

b. (5) In the case where $e\phi/k_B T \ll 1$, obtain the linear equation between the density and the potential for the two species.

c. (5) Write down the Poisson's equation and indicate where it came from.

d. (10) Show that taking into account the spherical symmetry of the problem, the equation for the potential is:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r}\frac{\partial \Phi}{\partial r} - \frac{1}{\lambda_D^2}\Phi = -\frac{Q}{\varepsilon_0}\delta(r)$$

and obtain the equation of λ_D as a function of the plasma parameters.

e. (5) Prove that the *Debye-Hückel* potential $\varphi(r) = \frac{Q}{4\pi\varepsilon_0 r^2} e^{-r/\lambda_D}$ is a solution to the above equation.

4- (40) Magnetic mirrors are used to confine charged particles in a limited volume. The gradient of magnetic field induction can result in reversing the direction of drift of a charged particle. The magnetic field induction is given by $B_z(z) = B_0(1 + (\gamma z)^2)$ where γ is a constant.

a. (5) Determine the expression of the magnetic field taking into account the axial symmetry of the problem.

b. (5) Show that, in general, and as a result of the magnetic field axial gradient, we have an average force with the form $F_z = -\frac{1}{2}\frac{mv_\perp^2}{B}\frac{\partial B_z}{\partial z}$. Comment on the minus sign.

c. (10) If the magnetic moment μ is defined as $F_z = -\mu\frac{\partial B_z}{\partial z}$, then prove that the magnetic moment is conserved, that is $\frac{d\mu}{dt} = 0$

Suppose that at $z = 0$, an electron has velocity $v^2 = 3v_\parallel^2 = 3/2v_\perp^2$.

d. (5) Describe qualitatively the electron motion.

e. (5) Determine the values of $z=Z$ where the electron is reflected.

f. (10) Write the equation of motion of the guiding center for the direction parallel to **B** and show that there is a sinusoidal oscillation. Calculate the frequency of the motion as a function of v.