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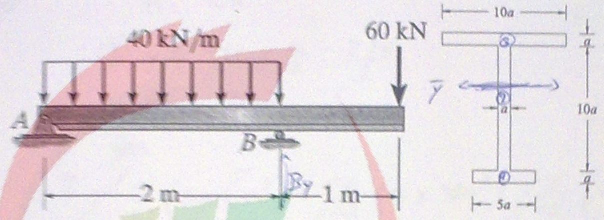
PROBLEM I: (25 points)

The beam is subjected to the loading shown.
Draw the shear and bending moment diagrams.
Determine its required cross-sectional dimension a (in mm), if the maximum allowable bending stress for the material is $\sigma_{allow} = 150$ MPa .



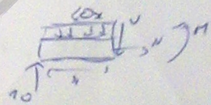
$$\sigma_{allow} = \frac{M_{allow}\, c}{I}$$

$$\circlearrowleft \sum M_A = 0 \quad -80 \times 1 + B_y \times 2 - 60 \times 3 = 0$$

$$B_y = 130 \text{ kN}$$

$$\sum F_y = 0 \quad A_y - 80 + 130 - 60 = 0$$

$$A_y = 10 \text{ kN}$$



$$\sum F_y = 0 \quad -V - 40x + 10 = 0$$

$$V = -40x + 10$$

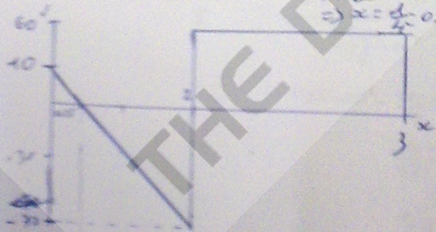
$$V = 0$$

$$\Rightarrow x = \frac{1}{4} = 0.25$$

$$\circlearrowleft \sum M = 0 \quad M + 20x^2 - 10x = 0$$

$$M = -20x^2 + 10x$$

$$M = 0 \Rightarrow x = 0,$$



① : $\bar{y} = \frac{a}{2}$

$A = 5a^2$

② $\bar{y} = 6a$

$A = 10a^2$

③ $\bar{y} = 11a + \frac{a}{2} = \frac{23a}{2}$

$A = 10a^2$

$\bar{y} = \frac{\Sigma yA}{\Sigma A} = \frac{\frac{5a^3}{2} + 60a^3 + 115a^3}{25a^2}$

$= \frac{\frac{355}{2}a}{25}$

$= \frac{355a}{\cancel{50}}$

$= \frac{71}{10}a = \boxed{7.1a}$

$$\cancel{I = \frac{1}{12}(5a \times a^3) + \frac{1}{12}(a \times 10a^3) + \frac{1}{12}(10a \times a^3)}$$

$$\cancel{= \frac{5a^4}{12} + \frac{10a^4}{12} + \frac{10a^4}{12} = \frac{25a^4}{12}}$$

$$I = \frac{1}{12}(5a \times a^3) + 5a^2\left(\frac{71}{10}a - \frac{a}{2}\right)^2 + \frac{1}{12}(a \times 10a^3) + 10a^2\left(\frac{71}{10}a - 6a\right)^2$$

$$+ \frac{1}{12}(10a \times a^3) + 10a^2\left(\frac{23a}{2} - \frac{71}{10}a\right)^2$$

$$= \cancel{\frac{5a^4}{12}} + \frac{25a^4}{12} + \frac{1089}{5}a^4 + \frac{121}{10}a^4 + \frac{968}{5}a^4$$

$$= \frac{5107}{12}a^4 \quad +$$

$$\sigma_{allow} = \frac{M_{max}c}{I} = \frac{60 \times \frac{71}{10}a}{\frac{5107}{12}a^4} = 150 \cancel{\times 10^6}$$

$$150 \times 10^6 \times \frac{5107}{12}a^3 = 60 \times \frac{71}{10}$$

$$\frac{5107}{12}a^3 = 2.84 \times 10^{-6}$$

$$a^3 = 6.67 \times 10^{-5}$$

$$\boxed{a = 0.188m}$$

$$\boxed{a = 188\ mm} \quad +$$

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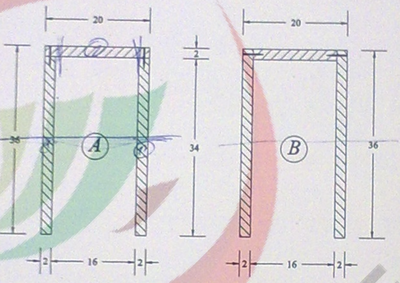
PROBLEM 2: (25 points)

Two wood beams (beams A and B) have the same outside dimensions (200 mm×360 mm) and the same thickness (t = 20 mm) throughout, as shown in the figure. Both beams are formed by nailing, with each nail having an allowable shear load of 250 N. the beams are designed for a shear force V = 3.2 kN.

a) What is the maximum longitudinal spacing s_A for the nails in beam A?
b) What is the maximum longitudinal spacing s_B for the nails in beam B?
c) Which beam is more economical (requiring a lesser number of nails)?



All dimensions are in cm.

$V = 3.2\ kN$ $\tau_{av} = 250 N$

$q' = \frac{VQ}{I}$ $q = \frac{q'}{2}$ $F = q \cdot s \Rightarrow \boxed{s = \frac{F}{q} = \frac{250}{q}}$

$\bar{y} = \frac{\Sigma yA}{\Sigma A} = \frac{2\times(17\times(2\times34)) + (35\times(2\times20))}{2\times((2\times34)) + (2\times20)} = \frac{3712}{108\times2} = \boxed{17.18}$ +

$I = 2\times\left(\frac{1}{12}(2)\times(34)^3 + (2\times34)(17.18-17)^2\right) + \left(\frac{1}{12}(20)(2)^3 + (2\times20)(35-17.18)^2\right)$

$= \cancel{13 kN}\ 2706.8$ +

$Q_A = \cancel{17.82\ (16\times2)}\ 8.41\times(2\times16.82) = 282.91$

$Q_B = 17.82(16\times2) = 570.24$ +

in A $q' = \frac{3200\times282.91}{2706.8} = 333.5$ $q = \frac{q'}{2} = 166.75$ $\boxed{s = \frac{250}{q} = 1.5\ cm}$ A

in B $q' = 674.1$ $q = 337.1$ $s = 0.7$ B

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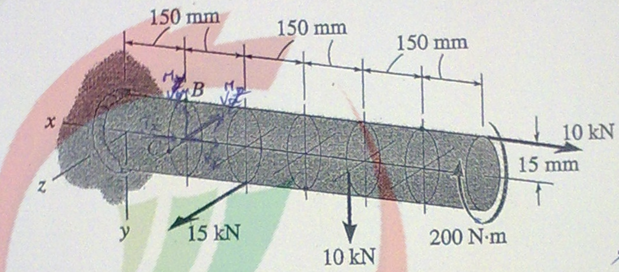
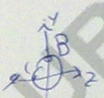
PROBLEM 3: (30 points)

The solid rod is subjected to the loading shown. Determine the state of stress at points *B*.

$I_y = I_z = \frac{\pi}{64}d^4$. Semicircular Area: $\bar{y} = \bar{z} = \frac{2d}{3\pi}$ (measured from the base of the semicircle)





$\Sigma F_x = 0 \qquad V_x + 10 = 0 \qquad \boxed{V_x = +10\ kN}$

$\Sigma F_y = 0 \qquad V_y - 10 = 0 \qquad \boxed{V_y = -10\ kN}$

$\Sigma F_z = 0 \qquad V_z - 15 = 0 \qquad \boxed{V_z = -15\ kN}$

$\Sigma M_x = 0 \qquad T_x - 15 \times 150 + 10 \times 150 = 0$

$T_x = \cancel{2250\ N/m} \qquad \boxed{-125\ N/m}$

$\Sigma M_y = 0 \qquad M_y - 15 \times 150 = 0 \qquad M_y = \boxed{2250\ N/m}$

$\Sigma M_z = 0 \qquad M_z + 10 \times 450 = 0 \qquad \boxed{M_z = -4500\ N/m}$

$A = \pi r^2 = \boxed{0.71\ m^2}$

$J = \frac{\pi}{2} c^4 = \boxed{8 \times 10^{-8}}$

$I_y = I_z = \frac{\pi}{64} d^4 = \cancel{2.3 \times 10^{-9}} \quad \boxed{3.97 \times 10^{-8}}$

$Q = \bar{y} A = \frac{4r}{3\pi} \times \frac{\pi r^2}{2} = \frac{2r^3}{3} = \boxed{0.01}$

$\sigma = \frac{N}{A} - \frac{M_z y}{I_z} + \frac{M_y z}{I_y} \qquad y = 15\ mm \quad z = 0$

$= \frac{-10}{0.71} + \frac{4500 \times 15 \times 10^{-3}}{3.97 \times 10^{-8}} = 1700 \times 10^{6}\ N$

$\tau = \frac{VQ}{It} \quad \frac{Tc}{J} \quad 10 \times 0.01 \quad + \quad -125 \times 15 \times 10^{-3}$

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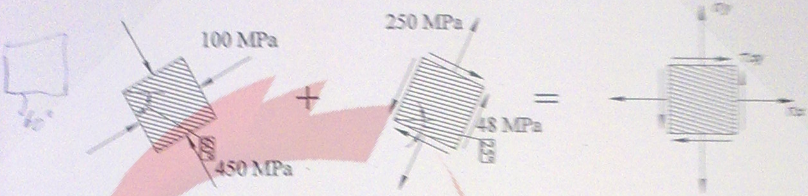
PROBLEM 4: **(20 points)**

A point on a thin plate is subjected to the two successive states of stress shown. Determine the resultant state of stress represented on the element oriented as shown on the right.



case ① $\sigma_x = -450$ MPa $\sigma_y = -100$ MPa $\tau_{xy} = 0$ $\theta = -60^\circ$ $2\theta = -120^\circ$

$$(\sigma_{x'})_1 = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \cancel{\tau_{xy}\sin 2\theta}$$

$= -275 + -175\cos 2\theta$

$= -275 + 87.5 = \boxed{-187.5 \text{ MPa}}$

$(\sigma_{y'})_1 = -275 - 87.5 = \boxed{-362.5 \text{ MPa}}$

$$(\tau_{xy})_1 = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \cancel{\tau_{xy}\cos 2\theta}$$

$= 151.55$ MPa

case ② $\sigma_x = 0$ $\sigma_y = 250$ MPa $\tau_{xy} = 48$ MPa $\theta = -25^\circ$ $2\theta = -50^\circ$

$(\sigma_{x'})_2 = 125 - 80.35 - 36.77 = 7.88$ MPa

$(\sigma_{y'})_2 = 125 + 80.35 + 36.77 = 242.12$ MPa

$(\tau_{xy})_2 = 95.75 + 30.85 = 126.6$ MPa

case 1 + case 2 = $(\sigma_{x'})_1 + (\sigma_{x'})_2 = -179.62$ MPa $= \boxed{179.62 \text{ MPa}}$

$(\sigma_{y'})_1 + (\sigma_{y'})_2 = -120.38$ MPa $= \boxed{120.38 \text{ MPa}}$

$(\tau_{x'y'})_1 + (\tau_{x'y'})_2 = 278.15$ MPa $= \boxed{278.15 \text{ MPa}}$