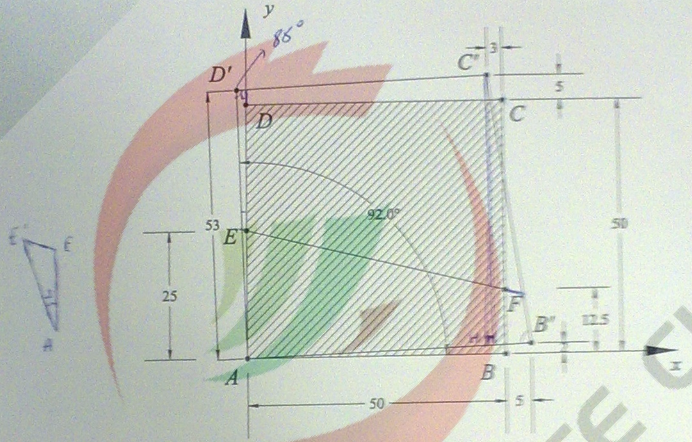
13

PROBLEM 1: (20 points)

The piece of rubber ***ABCD*** is originally square. After deformation its shape is defined by the polygonal line ***AB'C'D'***.
Determine the average shear strain γ_{xy} at point *A*.
Determine the average normal strains along the diagonal *AC* and line *EF*.
All dimensions are in millimeters (mm).



D'(−53 sin 2°, 53)   E(0, 25)
C'(47, 55)   F(50, 12.5)
B'(55, 2)   A(0, 0)

Shear strain at point A: $\gamma_{xy} = \pi/2 - \theta$
$= \pi/2 - (92 \cdot \frac{\pi}{180}) = \boxed{-0.035}$

$AC' = \sqrt{(47)^2 + (55)^2} = 72.35$ mm

$AC = \sqrt{50^2 + 50^2} = 70.71$ mm

Normal average strain $= \frac{72.35 - 70.71}{70.71} = \boxed{0.023}$

$EF = \sqrt{50^2 + 12.5^2} = 51.54$ mm

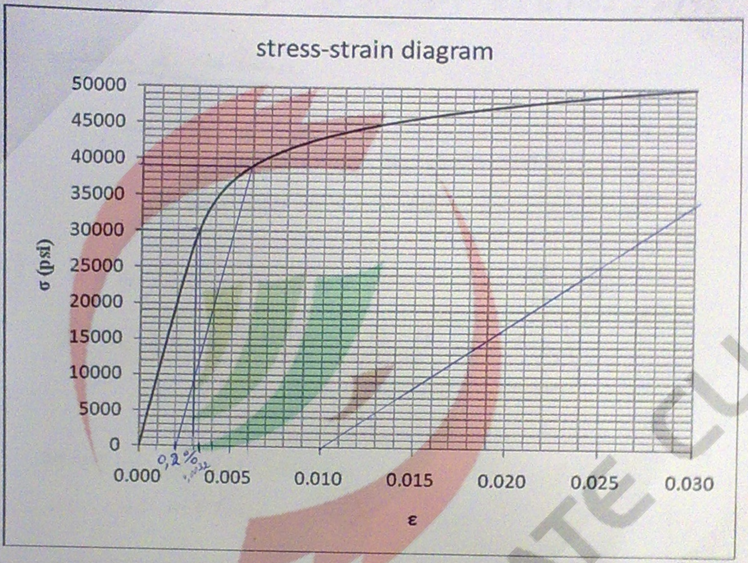
$C'D' = (50 - 3) + (53 \sin 2°) = 48.85$ mm

$C'B' = \sqrt{8^2 + 53^2} = 53.6$ mm

PROBLEM 2: (25 points)

12

A circular bar of aluminum alloy is 40 in. long. The stress-strain diagram for the material is shown in the figure.



Showing your points and construction on graphs and the associated calculations, determine the following quantities

1. Proportional Limit,
2. Modulus of Elasticity,
3. Yield Stress at 0.2% Offset,
4. The modulus of resilience for the material.
5. The permanent strain in the bar when it is loaded in tension to an elongation of 0.4 in., and then the load is removed.
6. If the bar is reloaded, what is the new proportional limit?

l = 40in

4) $\boxed{29\,000}$?

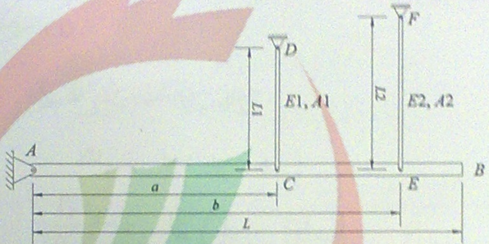
2) σ = 10000 ε = 0.0028

1/2 40000 × 0.006 = 120 +

PROBLEM 3: (25 points) 30

Rigid bar AB is pin-supported at A, and is intended to be connected to the two bars CD and EF, having the lengths and properties shown (E, and A). By mistake, the manufacturer made bar CD with a length greater than L_1 by an amount equal to δ_1.

What are the forces P_1 and P_2 in bars CD and EF respectively when they are connected to bar AB? Assume that Hooke's law applies for two bars CD and EF.



$\circlearrowleft \Sigma M_A = 0$

$-P_1 a + P_2 b = 0$

$\boxed{P_1 = \frac{b}{a} P_2}$ ①

by compatibility

$\frac{\delta_{1f}}{\delta_{2f}} = \frac{a}{b}$ $\quad \delta_{1f} = \frac{a}{b} \delta_{2f}$

or ~~Δl1~~ ~~and~~ $\delta_{2f} = \Delta l_2$ ~~and~~ $\delta_{1f} = \delta_1 - \Delta l_1$

$\Delta l_1 = \frac{a}{b} \Delta l_2$ $\quad$ ~~$\delta_1 = P_1 l$~~

$\boxed{\frac{P_1 L_1}{E_1 A_1} = \frac{a}{b} \frac{P_2 L_2}{E_2 A_2}}$ ②

$\boxed{P_1 = \left(\frac{a}{b}\right)\left(\frac{E_1 A_1}{E_2 A_2}\right)\left(\frac{L_2}{L_1}\right) P_2}$ ③

Replacing ① in ③

~~$\frac{b}{a} P_2$~~ and we have a, b, $E_1, A_1, E_2, A_2, L_1, L_2$ = cte

~~then we can find the mistake by calculating~~

~~$\delta_1 = \Delta l_1 = \frac{P_1 L_1}{A_1 E_1}$ and we already have L_1, A_1, E_1 and we had just find P_1 we will know the mistake elongation in the Bar CD.~~

$$\cancel{\delta_1 - \Delta L_1 = \Delta L_2 \, a/b}$$

$$\Delta l_2 = \frac{b}{a}\left(\delta_1 - \Delta l_1\right)$$

$$\frac{P_2 l_2}{A_2 E_2} = \frac{b}{a}\left(\delta_1 - \frac{P_1 L_1}{A_1 E_1}\right)$$

$$\boxed{P_2 = \left(\frac{b}{a}\right)\left(\frac{A_2 E_2}{L_2}\right)\left(\delta_1 - \left(\frac{L_1}{A_1 E_1}\right) P_1\right)} \quad (2)$$

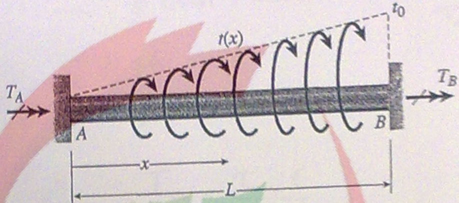
And we have $a, b, E_1, E_2, A_1, A_2, L_1, L_2$ and $\delta_1 = cte$ So when it is solved numerically we will work with equation (1) and (2) and we will find P_1 and P_2 then we can find δ_{1f} and δ_{2f}.

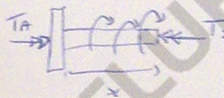
PROBLEM 4: (20 points) 5

15

A solid circular shaft ***AB***, of constant JG, with fixed supports at ends A and B is acted upon by the distributed torque t that varies linearly as shown in the figure.

a) Determine the reactive torques T_A and T_B in terms of t_0, JG and L.
b) For what distance x will the angle of twist ϕ be a maximum?
c) What is the corresponding angle of twist ϕ_{max}?





$+\circlearrowleft \sum M_x = 0 \qquad T_A - \int_0^L t(x)\,dx + T_B = 0$

$T_A - t_0 x \Big]_0^L + T_B = 0$

$T_A + T_B = T_0 L$ 2

$+\circlearrowleft \sum M_x = 0$

$T_A - \int_0^x t(x)\,dx - T_x = 0$

$T_x = T_A - \int_0^x t(x)\,dx$

$T_x = (T_0 L - T_B) - t(x)\,x$

$\phi = 0$

$\int_0^L \frac{T(x)\,dx}{JG} = 0$

$\frac{1}{JG}\int_0^L T(x)\,dx = 0$

$\frac{1}{JG}\int_0^L (T_0 L - T_B - t(x)\,x)\,dx = 0$

$\frac{1}{JG}\left(T_0 L x - T_B x - \frac{t(x)}{2}x^2\right)_0^L = 0$

$t_0 L^2 - t_B L - \frac{1}{2} t(x) L^2 = 0$

$t_B L = (t_0 - \frac{1}{2} t(x))L^2$

$\boxed{t_B = (t_0 - \frac{1}{2} t(x))L}$

$t_A = T_0 L - (t_0 - \frac{1}{2} t(x))L \qquad \boxed{t_A = \frac{1}{2} t(x) L}$