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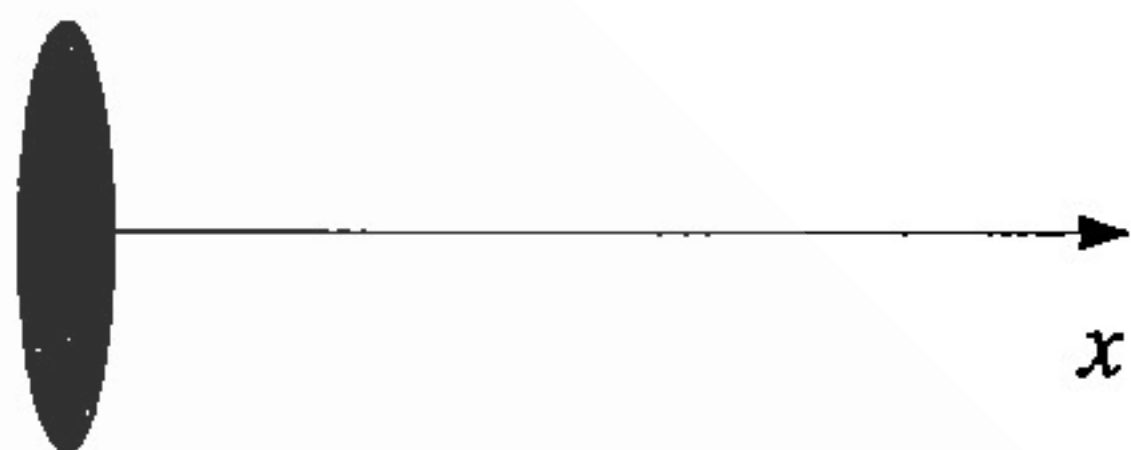
PHS 212 - Electricity & Magnetism

Faculty of Natural and Applied Sciences Midterm Exam II - Spring 2005

Department of Sciences

Duration: 100 minutes (1h40)

1. (5-G.7) An insulating disk of radius a is centered at the origin with its normal along the x -axis. It is given a charge density $\sigma = Br$.
- Find its total charge Q .
 - Find the potential $V(x)$ along the x -axis.
 - Verify that $V(x)$ has the expected behavior at large x .



2. (5-6.5) In the quark model, a proton consists of two "up" u quarks, each of charge $\frac{2}{3}e$, and a "down" d quark, of charge $-\frac{1}{3}e$. If they form an equilateral triangle of side $a = 10^{-15} \text{ m}$, find
- the potential at one of the up quarks due to the other 2 quarks,
 - the potential at the down quark due to the 2 up quarks, and
 - the total energy of all the quarks.

3. (6-7.1) A parallel-plate capacitor is charged up by a 6V battery, which is then removed. A dielectric with dielectric constant $\kappa = 5$ is slid between the plates. Give quantitative answers in terms of the ratio of each quantity before and after. a) What happens to the electric field within the dielectric? b) What happens to the voltage difference between the plates? c) What happens to the charge on the plates? d) What happens to the capacitance? e) What happens to the energy stored? Explain how energy is conserved.

4. (10-4.14) A loop of radius a , carrying current I counterclockwise as seen from $+x$, is in a magnetic field \vec{B} whose magnitude B is fixed, but whose direction always makes a flaring angle θ away from the axis of the loop. Draw and evaluate the force $d\vec{F}$ on an element $d\vec{s}$ that is at the top of the loop. Find the component of $d\vec{F}$ along the x -axis. Integrate to obtain the total force on the loop. Why is it only along the x -axis?

5. (10-6.4) An electron beam with a kinetic energy of 8 keV ($1 \text{ keV} = 1000 \text{ eV} = 1.6 \times 10^{-16} \text{ J}$) moves in a circle of radius 2.4 cm. Find
- its velocity,
 - the period of the motion,
 - the magnitude of the magnetic field it is in,
 - the magnetic force, and
 - the radial acceleration.

$m_e = 9.11 \times 10^{-31} \text{ kg}; e = 1.602 \times 10^{-19} \text{ C}$

Handwritten notes for problem 5:

$$v = \sqrt{2K/m_e}$$
$$T = 2\pi r/v$$
$$B = \frac{mv}{e r}$$
$$F = evB$$
$$a_r = v^2/r$$

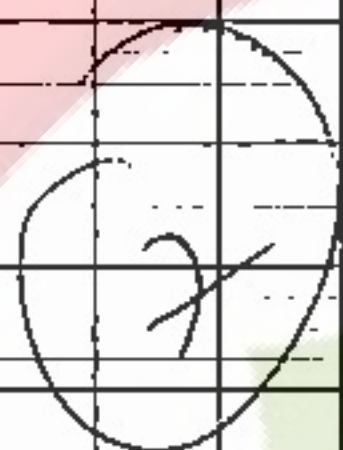


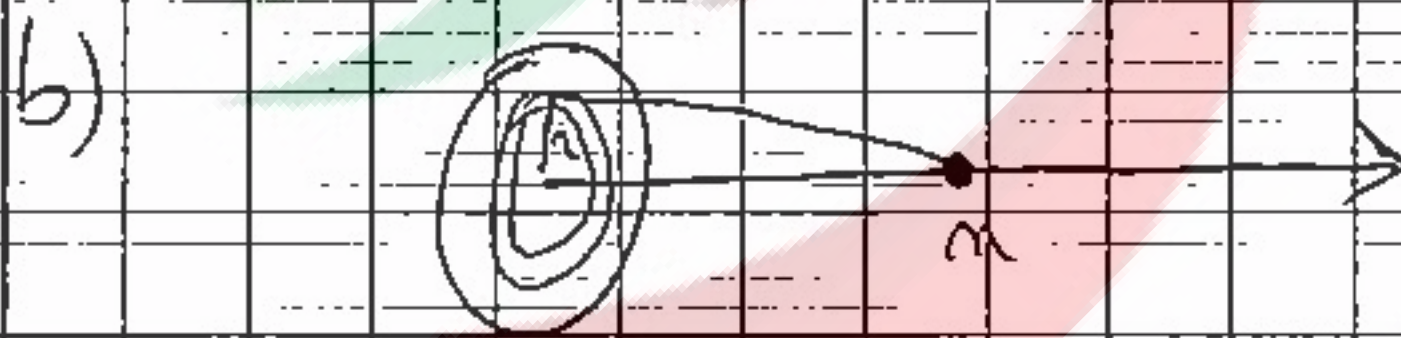
Prob 1:

$$dQ = \sigma dA \quad dA = 2\pi r dr$$

a) $Q = \int dQ = \int_0^a \sigma 2\pi r dr$

$= \int_0^a B r 2\pi r dr$
 $= 2\pi B \int_0^a r^2 dr = 2\pi B \left[\frac{r^3}{3} \right]_0^a$
 $= \frac{2\pi B a^3}{3}$





Differential element $dA = 2\pi r dr$
 use known that the potential

at a point on the axis passing through center of a charged circle is

$$V = \frac{kQ}{R} = \frac{kQ}{\sqrt{a^2 + z^2}}$$

So $dV = \frac{k dQ}{\sqrt{a^2 + z^2}} = \frac{k \sigma dA}{\sqrt{a^2 + z^2}}$

So $V(m) = \int \frac{k dQ}{\sqrt{a^2 + z^2}} = \int_0^a \frac{k \sigma 2\pi r dr}{\sqrt{a^2 + z^2}}$

$$= 2\pi K \int_0^a \frac{B a^2 da}{\sqrt{a^2 + m^2}} = 2\pi K B \int_0^a \frac{a^2 da}{\sqrt{a^2 + m^2}}$$

(10) Let $a = m \tan \theta$
 $\therefore \sqrt{a^2 + m^2} = \sqrt{m^2 \sec^2 \theta} = m \sec \theta$
 $\frac{da}{d\theta} = m \sec^2 \theta$
 $a^2 = m^2 \tan^2 \theta$

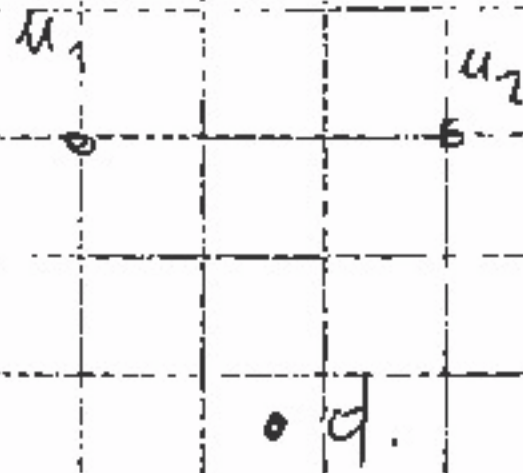
$$\int_0^a V(m) = 2\pi K B \int_0^{\theta} \frac{m^2 \tan^2 \theta \times m \sec^2 \theta}{m \sec \theta} d\theta =$$

$$= 2\pi K B \int_0^{\theta} m^2 \tan^2 \theta \sec \theta d\theta$$

~~$$= 2\pi K B m^2 \int_0^{\theta} \frac{\sin^2 \theta}{\cos^3 \theta} d\theta = 2\pi K B m^2$$~~

$$= 2\pi K B m^2 \int_0^{\theta} \sec \theta \tan^2 \theta \sec \theta d\theta$$

Prob. 2



(2) a)
$$V_{u_1} = \frac{k \left(\frac{1}{3} e \right)}{10^{-15}} + \frac{k \left(\frac{2}{3} e \right)}{10^{-15}} = \frac{1}{3} \frac{k e}{10^{-15}}$$

$$= 479.99 \cdot 10^3$$

(7) b)
$$V_d = \frac{\frac{2}{3} e k}{10^{-15}} + \frac{\frac{2}{3} e k}{10^{-15}} - \frac{4 \cdot e k}{3 \cdot 10^{-15}}$$

$$= 1.97 \cdot 10^6$$

(3) c)
$$U_T = \frac{1}{3} e V_{u_1} + \frac{2}{3} e V_{u_2} - \frac{1}{3} e V_d$$

$$= \frac{1}{3} e V_{u_1} + \frac{2}{3} e V_{u_2} - \frac{1}{3} e V_d$$
~~$$= 639.98 \cdot 10^3 +$$~~

$$= 479.99 \cdot 10^3 \times \left(\frac{4}{3} e \right) - \frac{1}{3} e (1.97 \cdot 10^6)$$

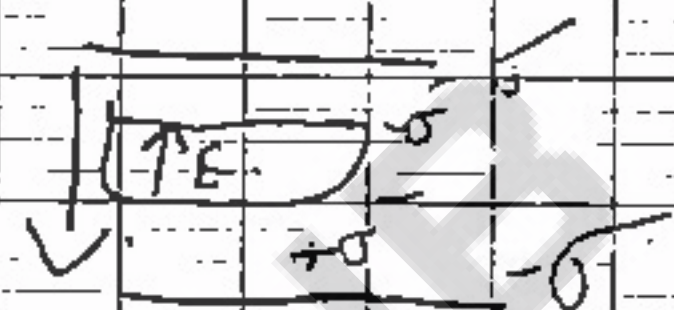
$$= 5.379304 \cdot 10^{16}$$

Prob. 3:

a) Under the dielectric the electric field will be minimized

because the true surface of the dielectric will be charged

So E will be $E_0 - E'$



$$E_0 = K E_d \quad (K > 1)$$

$$E_0 = 3 E_d$$

b) $\Delta V_0 = E_0 d = K E_d \times d = K \Delta V$

so for $\frac{\Delta V_0}{\Delta V} = 3$ after

The potential will be less under the dielectric

c) The charges on the plates will be unchanged why??

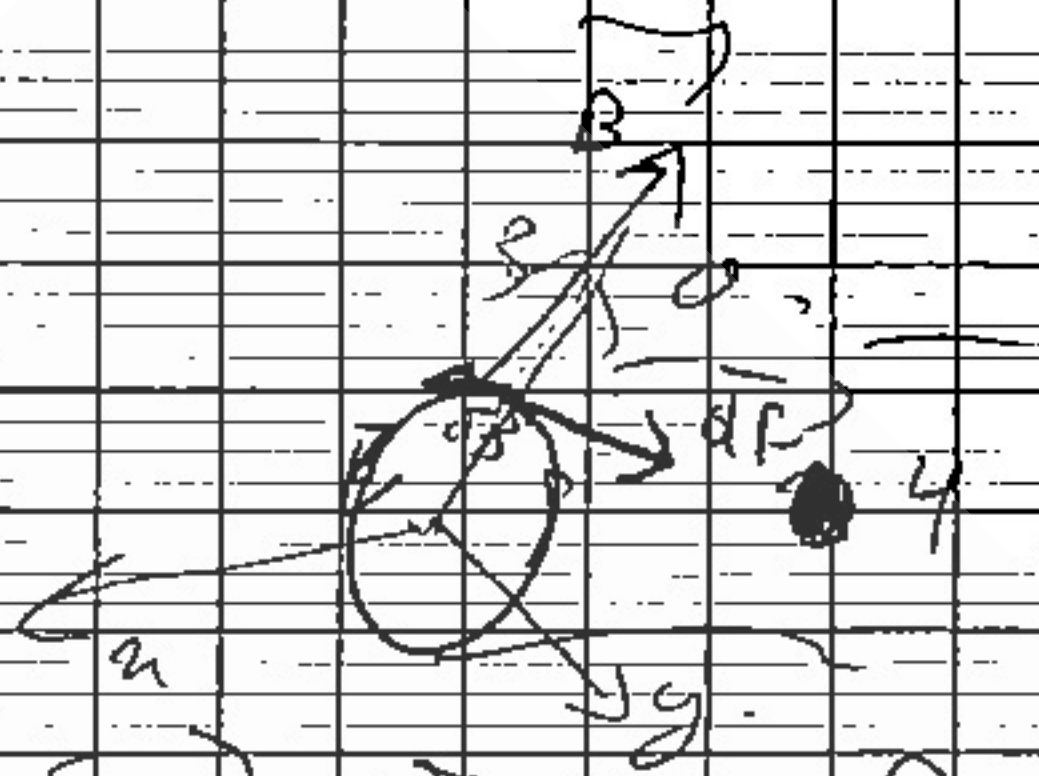
d) The capacitance will be greater because ΔV is minimized and

d is constant

$$C = K C_0$$

$$C_0 = 3$$

Prakt. 4.



$$dF = \int dA \cdot R \rightarrow \int dA \cdot B$$

$$= \int dA \cos(\theta) \cdot B \quad (\text{where } \theta = \text{angle between } dA \text{ and } B)$$

$$= \int dA \cos(\theta) \cdot B \quad \text{or} \quad \int dA \cdot B$$

$$dF = V \left(\int dA \cos(\theta) + \int dA \sin(\theta) \right) \sin(\theta)$$

$$= \int dA \cos(\theta)$$

always \times over $dA \sin(\theta) = \int B \cdot dA \sin(\theta)$

~~$$F = \int \left(\int dA \cos(\theta) \cdot B + \int dA \sin(\theta) \cdot B \right) \sin(\theta)$$~~

~~$$\int dA \cos(\theta) \cdot B + \int dA \sin(\theta) \cdot B$$~~

always along x axis because
of the symmetry

$$\begin{aligned}
 \vec{dF}' &= \vec{dS}' \times \vec{B}' \\
 &= \vec{I} (\vec{ds}' \times \vec{B}') \\
 &= \vec{I} (\cos \theta \vec{ds} + \sin \theta \vec{ds}') \times \vec{B}' \\
 &= \vec{I} (\cos \theta \vec{ds} + \sin \theta \vec{ds}') \times (-B \cos \theta \hat{n} + B \sin \theta \hat{s})
 \end{aligned}$$

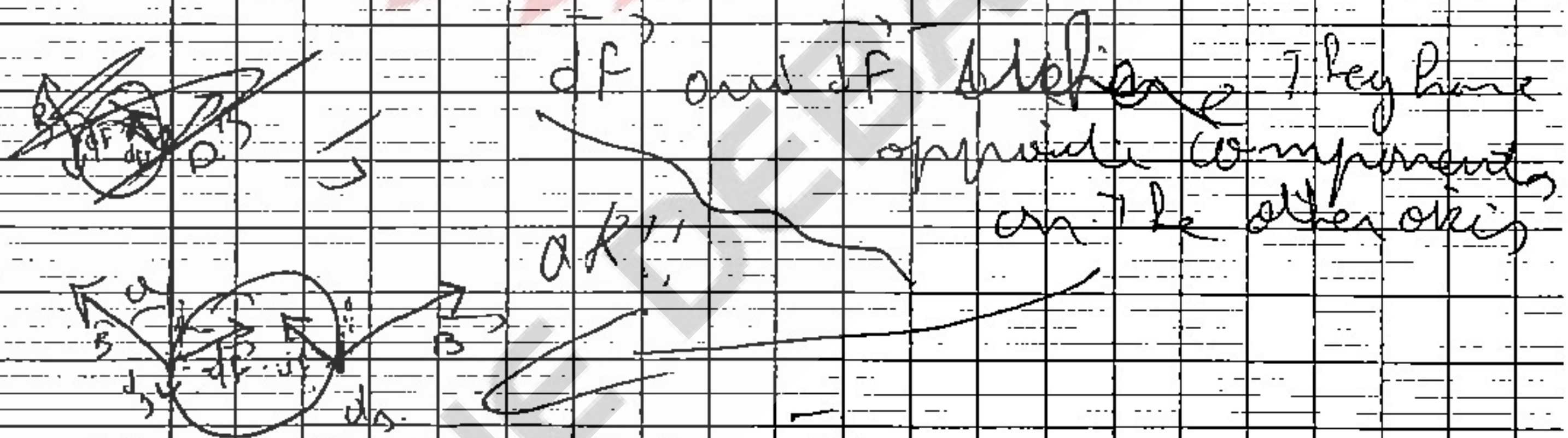
$$\begin{aligned}
 \vec{A} = \int \vec{dF}' &= \vec{I} \int \vec{B} \cdot \vec{ds} \hat{n} \\
 &= 2\pi a \int B \sin \theta
 \end{aligned}$$

It's only about the max because \vec{ds}' is in the yz plane and \vec{ds} is in the xz plane

$$\vec{ds}' = \vec{I} \times \vec{r} \times \vec{e}_z$$

Because for the ~~diagonal~~ \vec{ds}

There's another \vec{ds} there.



Prob: 5.

a) $KE = \frac{1}{2} m v^2 = 1 \text{ mV}^2 = 8 \times 10^{-12} \text{ J}$
 $v = 53.01 \times 10^6 \text{ m/s}$

b) $T = \frac{2\pi a}{v} = \frac{2\pi \times 2.4 \times 10^{-7}}{53.01 \times 10^6} = 2.845 \times 10^{-13} \text{ s}$

c) $\vec{F} = m \vec{a} = q \vec{v} \times \vec{B}$ ($v \perp B$)

$F_0 = \frac{m v^2}{r} = q v B$

$B = \frac{m v^2}{r q v}$

d) $\vec{F} = q \vec{v} \times \vec{B}$

$F = q v B$

$= 4.04 \times 10^{-13} \text{ N}$

e) $a_r = \frac{v^2}{r} = 1.77 \times 10^{17} \text{ m/s}^2$