Dr Sabra

OPEN BOOK

25

Notre Dame University-Louaize
Faculty of Natural & Applied Sciences
Department of Sciences

PHS 212 – Electricity & Magnetism
Exam I - Fall 2007
Duration: 30 minutes (1/2h)

Name:

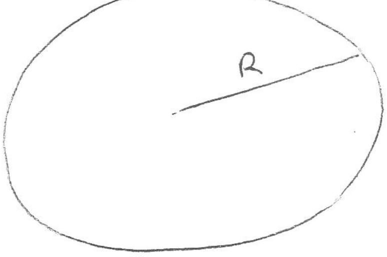
ID:

1) A sphere of radius R carries a volume charge distribution $\rho(r) = \rho_0 \left(\frac{r}{R}\right)^3$, where ρ_0 is a positive constant.

   a. Find the electric field, both magnitude and direction, for $r < R$, and $r > R$.

   b. Find the potential for $r < R$, and $r > R$. (Take $V \to 0$ when $r \to \infty$)

$\rho(r) = \rho_0 \left(\frac{r}{R}\right)^3$



a) for $r < R$:

Gauss's Law: $\int \vec{E} \cdot \hat{n}\, dA = \frac{Q_{enc}}{\varepsilon_0}$ ✓ what geometry for the surface?

(8)

$Q_{encl} = \int \rho(r)\, dV \qquad dQ = \rho\, dV \Rightarrow dV = 4\pi r^2 dr$ ✓

$= \int_0^r \rho_0 \left(\frac{r}{R}\right)^3 4\pi r^2 dr$

$= \int_0^r \frac{\rho_0}{R^3} 4\pi\, r^5 dr' = \frac{\rho_0}{R^3} 4\pi \left[\frac{r^6}{6}\right]_0^r$

$= \frac{\rho_0\, 4\pi r^6}{6R^3}$ ✓

$E\, 4\pi r^2 = \frac{\rho_0\, 4\pi r^6}{6R^3 \varepsilon_0} \Rightarrow E(r<R) = \frac{\rho_0 r^4}{6R^3 \varepsilon_0}$ ✓ (radially out) ✓

For $r > R$:

$\int \vec{E} \cdot d\vec{A} = \frac{q_{encl}}{\varepsilon_0}$ ✓

$E\, 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E(r>R) = \frac{Q}{4\pi r^2 \varepsilon_0}$ ✓ $Q = ?$

(2)

b) for $r < R$: $V(R) = \frac{kQ}{R}$ ✓

$$V(r) - \frac{kQ}{R} = -\int_R^r \frac{\rho_0 r^4}{6R^3\varepsilon_0} dr$$ ✓

$$V(r) - \frac{kQ}{R} = -\frac{\rho_0}{6R^3\varepsilon_0}\left[\frac{r^5}{5}\right]_R^r$$ ✓

$$= -\frac{\rho_0}{6R^2\varepsilon_0}\left[\frac{r^6}{6} - \frac{R^5}{5}\right]$$ ✓

(8)

$$= -\frac{\rho_0 r^6}{36R^2\varepsilon_0} + \frac{\rho_0 R^3}{30\varepsilon_0}$$ ✓

$$V(r) = \frac{kQ}{R} - \frac{\rho_0 r^6}{36R^2\varepsilon_0} + \frac{\rho_0 R^3}{30\varepsilon_0}$$ ✓

for $r > R$:

$$V(r) - V_\infty = -\int_\infty^r \vec{E}\cdot d\vec{r}$$ ✓

$$V(r) - 0 = -\int \frac{Q}{4\pi\varepsilon_0 r^2} dr$$ ✓

(7)

$$V(r > R) = \frac{kQ}{r}$$ ✓

b) ~~$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$~~

~~$C_1 = \frac{\varepsilon_0 A}{d} = 0$~~ ~~$C_2 = \frac{k \varepsilon_0 A}{t}$~~ ~~$C_3 = \frac{\varepsilon_0 A}{(d-t)}$~~

b) $C_{eq} = ~~C_1~~ + C_2 + C_3$

$C_2 = \frac{k \varepsilon_0 A}{t}$ (5) , $C_3 = \frac{\varepsilon_0 A}{(d-t)}$ (5)

$C_{eq} = \varepsilon_0 A \left(\frac{k}{t} - \frac{1}{d-t} \right)$

c) Because the charge density induced in the dielectric weakens the field of the charge on the plates, for the dielectric the charge density σ_{diel} on the plates must be larger than the charge density σ_{air} without the dielectric.

~~$\sigma_{air} = \frac{\sigma_{diel}}{K}$~~