NDU | **PHS 212 - Electricity & Magnetism**

Faculty of Natural and Applied Sciences | **Midterm Exam II -**

Department of Sciences | **Duration: 100 minutes (1h40)**

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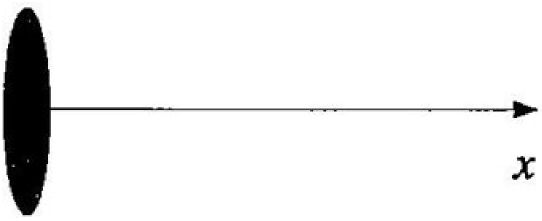
1. (5-G.7)An insulating disk of radius a is centered at the origin with its normal along the x-axis. It is given a charge density $\sigma = Br$.
   a. Find its total charge Q.
   b. Find the potential $V(x)$ along the x-axis.
   c. Verify that $V(x)$ has the expected behavior at large x.



2. (5-6.5) In the quark model, a proton consists of two "up" u quarks, each of charge $\frac{2}{3}e$, and a "down" d quark, of charge $-\frac{1}{3}e$. If they form an equilateral triangle of side $a = 10^{-15} m$, find
   a. the potential at one of the up quarks due to the other 2 quarks,
   b. the potential at the down quark due to the 2 up quarks, and
   c. the total energy of all the quarks.

3. (6-7.1) A parallel-plate capacitor is charged up by a 6V battery, which is then removed. A dielectric with dielectric constant $\kappa = 5$ is slid between the plates. Give quantitative answers in terms of the ratio of each quantity before and after. a) What happens to the electric field within the dielectric? b) What happens to the voltage difference between the plates? c) What happens to the charge on the plates? d) What happens to the capacitance? e) What happens to the energy stored? Explain how energy is conserved.

4. (10-4.14) A loop of radius a, carrying current I counterclockwise as seen from $+x$, is in a magnetic field $\vec{B}$ whose magnitude B is fixed, but whose direction always makes a flaring angle θ away from the axis of the loop. Draw and evaluate the force $d\vec{F}$ on an element $d\vec{s}$ that is at the top of the loop. Find the component of $d\vec{F}$ along the x-axis. Integrate to obtain the total force on the loop. Why is it only along the x-axis?

5. (10-6.4) An electron beam with a kinetic energy of 8 keV ($1\,keV = 1000\,eV = 1.6\times10^{-16} J$) moves in a circle of radius 2.4 cm. Find
   a. its velocity,
   b. the period of the motion,
   c. the magnitude of the magnetic field it is in,
   d. the magnetic force, and
   e. the radial acceleration.

   $m_e = 9.11\times10^{-31} kg ; e = 1.602\times10^{-19} C$

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Prob 1:

$dQ = \sigma\, dA \qquad dA = 2\pi r\, dr$

a) $Q = \int dQ = \int_0^a \sigma\, 2\pi r\, dr$

$= \int_0^a Br\, 2\pi r\, dr$

$= 2\pi B \int_0^a r^2\, dr = 2\pi B \left[\frac{r^3}{3}\right]_0^a$

$= 2\pi B \frac{a^3}{3}$

(7)

b)

Differential element $dA = 2\pi r\, dr$

we know that the potential

on a point on the axes passing on the center of a charged circle is

$V = \frac{KQ}{R} = \frac{KQ}{\sqrt{a^2+x^2}}$

So $dV = \frac{K\,dQ}{\sqrt{R^2+x^2}} = \frac{K\sigma\, dA}{\sqrt{R^2+x^2}}$

So $V(x) = \int \frac{K\,dQ}{\sqrt{R^2+x^2}} = \int_0^a \frac{K\sigma\, 2\pi r\, dr}{\sqrt{r^2+x^2}}$

$$= 2\pi K \int_0^a \frac{B r^2 \, dr}{\sqrt{a^2 + x^2}} = 2\pi K B \int_0^a \frac{r^2 \, dr}{\sqrt{r^2 + x^2}}$$

(10) let $r = x \tan\theta$

$\therefore$ So $\sqrt{r^2 + x^2} = \sqrt{x^2 \sec^2\theta} = x \sec\theta$.

$\frac{dr}{d\theta} = x \sec^2\theta$

$r^2 = x^2 \tan^2\theta$

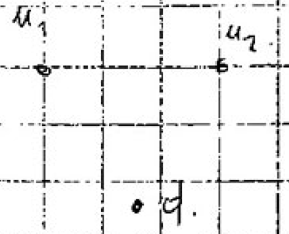
$$\text{So } V(x) = 2\pi K B \int \frac{x^2 \tan^2\theta \times x \sec^2\theta \, d\theta}{x \sec\theta}$$

$$= 2\pi K B \int x^2 \tan^2\theta \sec\theta \, d\theta.$$

~~$= 2\pi K B x^2 \int \frac{\sin^2\theta}{\cos^3\theta} d\theta = 2\pi K B x^2$~~

$$= 2\pi K B x^2 \int \sec\theta \tan\theta \times \tan\theta \, d\theta.$$

Prob. 2



a) $V_{u_1} = \frac{K(\frac{1}{3}e)}{10^{-15}} + \frac{K(\frac{2}{3}e)}{10^{-15}} = \frac{1}{3}\frac{Ke}{10^{-15}}$

(7) ✓

$= 479.99 \cdot 10^{3}$

b) $V_d = \frac{\frac{2}{3}eK}{10^{-15}} + \frac{\frac{2}{3}eK}{10^{-15}} = \frac{4 \cdot eK}{3 \cdot 10^{-15}} =$

(7) ✓

$1.91 \cdot 10^{6}$

c) $U_T = \frac{1}{3}e V_{u_1} + \frac{2}{3}e V_{u_2} - \frac{1}{3}e V_d$

(3)

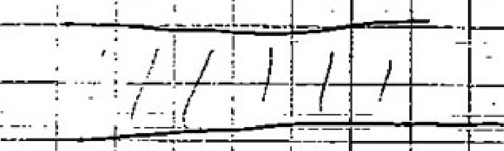
$= q_1 V_{u_1} + q_2 V_{u_2} + q_3 V_d$

~~$= \ldots 634.98 \cdot 10^{3} +$~~

$= 479.99 \cdot 10^{3} \times (\frac{1}{3}e) - \frac{1}{3}e(1.91 \cdot 10^{6}$

$= 5.379304 \cdot 10^{-16}$

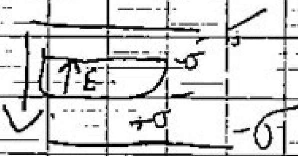
Prob. 3.



a) inside the dielectric the electric field will be minimized

because the two surfaces of the dielectric will be charged

So E will be $E_0 - E'$



$E_0 = K E_d$ $(K > 1)$

$E_0 = 3 E_d$

b) $\Delta V_0 = E_0\, d = K E_d \times d$

$= K \Delta V_{after}$

before $\frac{\Delta V_0}{\Delta V} = 5$

The potential will be less inside the dielectric

c) The charge on the plane will be unchanged why???

d) The capacitance will be greater because ΔV is minimized and Q is constant.

$C = K C_0$

$\frac{C}{C_0} = 3.$

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Prob 3-

e) $u = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}C_0(\Delta V)^2$

$= \frac{1}{2}C\kappa(\Delta V)^2 \quad \kappa\frac{1}{2}C(\Delta V)^2$

$\kappa \quad \kappa u$

So $\frac{u_0}{u} = 5$ energy is less because C ↑

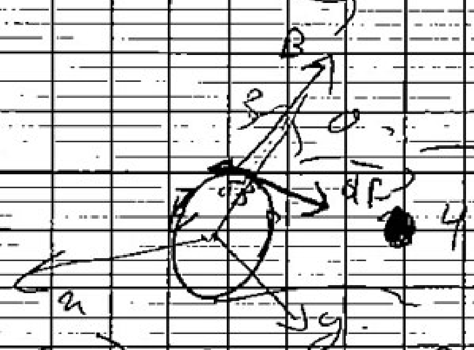
energy is ↑ to zero

Prob 4 -

$d\vec{F} = I\,d\vec{s} \times \vec{B} = I\,ds\,B\sin\theta$

dF only $\vec{e}$

Prob. 4



$d\vec{F} = I\, d\vec{s} \times \vec{B} = I\, d\vec{s}\,.\,\vec{B}$

$= I\, ds\,(-\hat{j})\,.\,B(\cos\theta\,\hat{x} + \sin\theta\,\hat{z})$ ✓

$= I\, ds\, B \cos\theta\, \hat{z} - I\, ds\, B \sin\theta\, \hat{x}$

$dF = \sqrt{(I\, ds\, B\cos\theta)^2 + (I\, ds\, B)^2 \sin^2\theta}$

$= B I\, ds$

along x axis $dF_x = -I B\, ds \sin\theta$

~~$\vec{F} = \int (I\, ds\, B\cos\theta\, \hat{z} - I\, ds\, B \sin\theta\, \hat{x})\, ds$~~

~~$= I B \cos\theta\, \hat{z}\, 2\pi a - 2\pi a\, I B \sin\theta\, \hat{x}$~~

~~always along x axis because of the sym~~

$$d\vec{F} = I\, d\vec{s} \times \vec{B}$$

$$= I(ds\,\vec{j} + ds\,\vec{k}) \times \vec{B}$$

$$= I(ds\,\vec{j} + ds\,\vec{k}) \times (-B\cos\theta\,\vec{n} + B\sin\theta\,\vec{j})$$

$$F = \int dF = \int I\,B\,ds\sin\theta$$

$$= 2\pi a I B\sin\theta$$

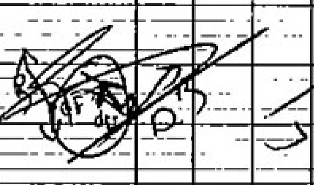
It's only about the x axis

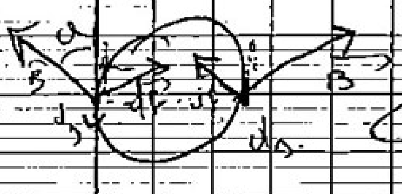
~~because $d\vec{F}$ be $d\vec{s}$ // to the yz plane and $\vec{B}$ is // to xz plane~~

~~So F is in the xz~~

Because for the ~~ds~~ ~~diameter~~ ds like a other ds ~~diam~~

$d\vec{F}$ and $d\vec{F}'$ ~~delete~~ They have opposite components on the other axis





Prob. 5.

a) $KE = \cancel{2\,KeV} = \frac{1}{2} m v^2 = \cancel{3.2 \times 10^{-16}}$ J

$v = 53.01 \cancel{\times 10^{6}}$ m/s (4)

b) $T = \frac{2\pi a}{v} = \frac{2\pi \times 2.4 \times 10^{-?}}{53.01 \times 10^{6}} = \cancel{2.84} 5 \times 10^{-9}$ s (4)

c) $\vec{F} = m\vec{a} = q\vec{v} \times \vec{B}$ ($v \perp B$)

So $m\frac{v^2}{R} = |q| v B$

$B = \frac{m v^2}{R q v} = \cancel{0.012}$ (4)

d) $\vec{F} = q\vec{v} \times \vec{B}$

$F = q v B$

$= 1.01 \times 10^{-13}$ (4)

e) $a_r = \frac{v^2}{R} = 1.17 \times 10^{17}$ (4)