NDU
Faculty of Natural and Applied Sciences
Department of Sciences

PHS 212 - Electricity & Magnetism
Midterm Exam - Spring 2005
Duration: 100 minutes (1h40)

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1. **Calculators and your original textbook are allowed.**
2. **It is strongly advised that you read your exam from A to Z before you start working on any of the problems. I do not expect you to solve the problems in sequence; feel free to start with the one you like most! This will help boost your confidence.**
3. **You can get partial credit for all the problems as long as you detail your answers.**
4. **All other rules and regulations pertaining to exams stated in NDU the catalog are considered an integral part of this document.**
5. **You are not allowed to borrow anything during the exam.**
6. **You are not allowed to talk with anyone during the exam. Any such behavior will be considered a cheating attempt, even if you are talking with someone sitting for another exam.**
7. **You paper will be taken at the first cheating attempt. Do not expect a warning!**
8. **This page should be signed, teared off and put on the side of the table before you start the exam. Failure to do so will result in a 15% decrease of your grade.**

I have read and understood the rules and regulations governing this exam.

Louaize, April 19, 2005

Problem 1	25 /25	*Problem 3*	25 /25
Problem 2	13 /25	*Problem 4*	25 /25

88

Good Luck!

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PHS 212 - Electricity & Magnetism
Midterm Exam I - Spring 2005
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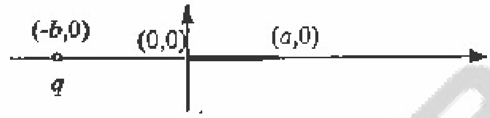
1. Two point charges sum to $-5\mu C$ C. At a separation of 2 cm, they exert a force of 80 N on each other. Find the two charges when
   a. The forces are attractive
   b. The forces are repulsive

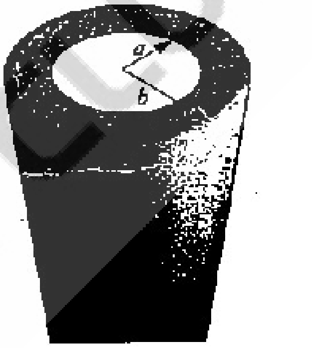
2. A rod of length a whose ends are at (0,0) and (a,0) has a charge density $\lambda = (\frac{Q_0}{a^2})x$ (see figure)

   a. Find the total charge Q on the rod.
   b. Find the force on a charge q at ($-b$,0).
   c. Verify that the force has the correct limit as $b \to \infty$

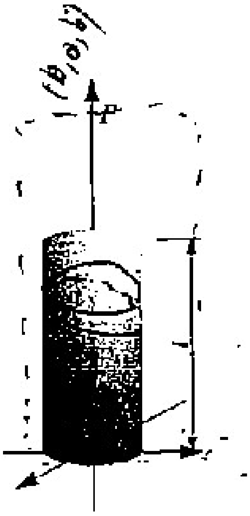


$K\frac{qQ_0}{a^2}\int \frac{\cancel{K}\cancel{A}\; x\,dx}{(x+b)^2}$

3. An infinetly long cylindrical shell of inner radius a and outer radius b carries a uniform volume charge density ρ. Using Gauss' Law, find the electric field for $r<a$, $a<r<b$, and $r>b$.



4. A cylinder, length L and radius R, carries a uniform charge density ρ. Its base is in the z=0 plane (see figure). Find the electric field (magnitude and direction) at the point P on the z-axis (the axis of the cylinder).



$\frac{KqQ_0}{a^2}\int \frac{1}{(x+b)} - \frac{b}{(x+b)^2}$

$\frac{KqQ_0}{a^2}\left[\ln(x+b)+\frac{b}{x+b}\right]_0^L$ $V = 2\,d\theta\, dr\, dh$

$\frac{KqQ_0}{a^2}\left[\ln(L+b)+\frac{b}{L+b} - \ln b - 1\right]$. $\frac{KqQ_0}{a^2}\left[\frac{L}{L+b}\right)$

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Exam 1

Grade : 88/100

Course N°: PHS 212 Section : Dr. Kreidy Date :

Ex 6

N°1:

$Q_1 + Q_2 = -5 \mu C = -5 \cdot 10^{-6} C$

$r = 2 cm = 2 \cdot 10^{-2} m$

$|F| = 80$

a) If the two charges are attractive so

$Q_1 > 0$ and $Q_2 < 0$

$F = \frac{K Q_1 Q_2}{r^2} = 80$

$80 = \frac{-K Q_1 (-5 \cdot 10^{-6} - Q_1)}{(2 \cdot 10^{-2})^2}$

$0.032 = +5 \cdot 10^{-6} K Q_1 + K Q_1^2$

$8.9875 \cdot 10^9 Q_1^2 - 44937.5 Q_1 - 0.032 = 0$

~~$Q_1 = 4.13$~~ $Q_2 = 5.65 \cdot 10^{-6} C$

$Q_2 = -8.6 \cdot 10^{-7}$ $Q_1 = 6.52 \cdot 10^{-7} C$

2.5

please improve your handwriting

b) If the force are repulsive so

Q_1 and $Q_2 > 0$ or Q_1 and $Q_2 < 0$

only the possibility Q_1 and $Q_2 < 0$ acceptable since

There min is CO

$$\delta O = \frac{K |q_1| |q_2|}{r^2}$$

$$0.032 = 5.10^{-6} K q_1 - K q_1^2$$

$$0.032 = 44937.5\, q_1 - 8.9875 \times 10^9 q_1^2$$

$$-8.9875 \times 10^9 q_1^2 + 44937.5\, q_1 - 0.032 = 0$$

$$\delta O = \frac{K |q_1| (5.10^{-6} - |q_1|)}{(2.10^{-2})^2}$$

$$K q_1^2\, 5.10^{-6} - K q_1 = 0.032$$

$$+8.9875 \times 10^9 q_1^2 - 44937.5\, q_1 + 0.032 = 0$$

$q_1 = 8.6 \cdot 10^{-7}\, C$ $\quad q_2 = 4.139 \cdot 10^{-6}$

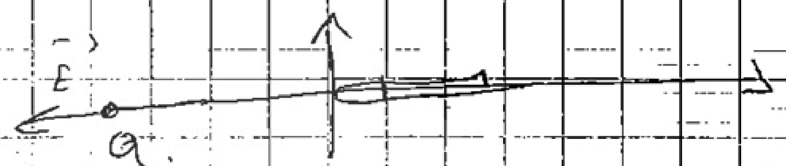
Problem 2:

$$Q = \int_0^Q dQ = \int_0^a \lambda\, dx$$

$$= \int_0^a \frac{(Q_0)x}{a^2}\, dx = \frac{Q_0}{a^2}\left[\frac{x^2}{2}\right]_0^a = \frac{a^2 Q_0}{2 a^2}$$

$$= \frac{Q_0}{2}$$

b)



$$dE = \frac{K\,dQ\,q}{x^2} = \frac{Kq\lambda\,dx}{(b+x)^2}$$

$$F = \int_0^a \frac{Kq\frac{Q}{a}\,dx}{(b+x)^2} = K\lambda q \int \frac{dx}{(b+x)^2}$$

3

$$= K\lambda q\left[\frac{-1}{b+x}\right]_0^a = K\lambda q\left[\frac{1}{b+a} - \frac{1}{b}\right]$$

~~$\vec{E} = q\vec{E}$~~

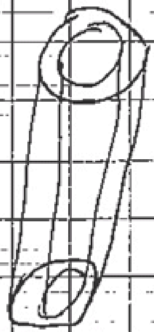
$$\cancel{F = K\lambda q\left(\frac{1}{b+a} - \frac{1}{b}\right)}$$

c) If $b \to \infty$

$$F = K\lambda q\left[\frac{1}{a}\right] = \frac{KQq}{a\,a} = \frac{KQq}{a^2}$$

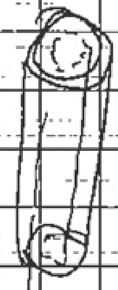
So the rod will ~~be~~ behave like a particle charge point

3.)



If $r < a$

Gaussian Surface is a cylinder with radius $r < a$ concentric with this shell



Gauss law: The field is radial and is the same on the Gaussian Surface.

(8) The flux in the upper and down surface is 0 since $\vec{E} \perp \vec{n}$

Gauss law

$\phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$

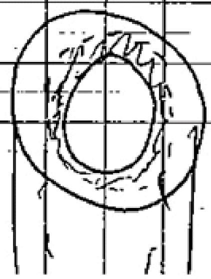
if $r < a$ q_{enc} is 0 so $\phi = 0$

since $\vec{E}$ and $d\vec{A}$ same direction

$\phi = E \oint dA = EA = 0$ so $E = 0$

If $a < r < b$:

let as Gaussian Surface is the cylinder Concentric with the shell and ~~the~~ with $a < r < b$.



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Name : Georges Barber   I.D. No :
Course No :   Section :   Date :

$\phi = \int E\, dA = \frac{Q_{enc}}{\varepsilon_0}$

$Q_{encl} = \rho V$

$V = \pi r^2 l \quad \pi a^2 l \quad = \pi l (r^2 - a^2)$

~~$\phi = E \oint dA = E(\pi r^2 - \pi a^2)$~~

~~If a > b~~

9 ✓ So $\phi = E \oint dA = E(2\pi r l) = \frac{Q_{en}}{\varepsilon_0}$

$E = \frac{Q_{en}}{2\pi r l\, \varepsilon_0} = \frac{\rho \cancel{\times} \pi l (r^2 - a^2)}{2\pi r l\, \varepsilon_0}$

$= \frac{\rho (r^2 - a^2)}{2 \varepsilon_0 r} = \frac{1}{2} \frac{\rho (r^2 - a^2)}{r \varepsilon_0}$

I r > a Same principle with Gaussian surface a cylinder with radius > b

$\oint E\, dA = \frac{Q_{enc}}{\varepsilon_0}$

$Q_{enc} = \rho V = \rho(\pi b^2 l - \pi a^2 l)$

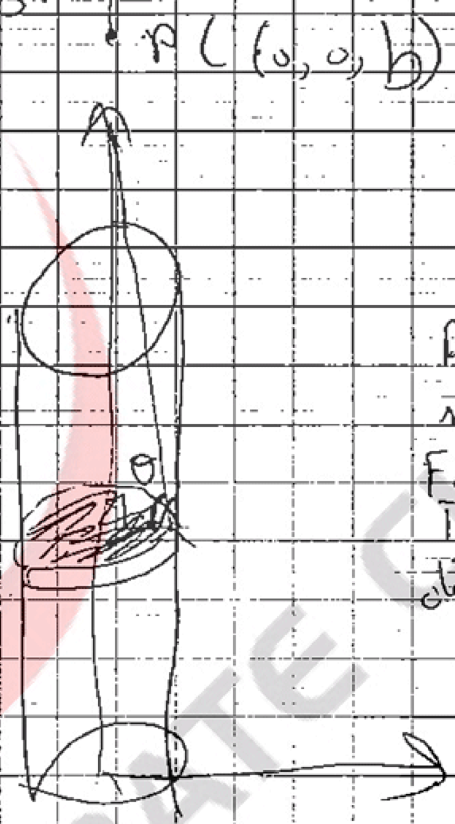
$\phi = E\, 2\pi r l$

8 So $E \cdot 2\pi r l = \dfrac{\rho(\pi b^2 l - \pi a^2 l)}{\varepsilon_0}$ ✓

$E = \dfrac{\rho(b^2 - a^2)}{2 r \varepsilon_0}$

$P((0,0,b))$

Problem 4:



Due to the symmetry the field will take the direction of y axis.

$d\vec{E} = \dfrac{K\,dQ}{z^2}\,\vec{j}$

let us take the disk of volume:

$dV = \pi R^2 dz$ as a differential element.

So the known from page 120:

$d\vec{E} = 2\pi K \sigma \,(b-z)\left(\dfrac{1}{(b-z)} - \dfrac{1}{\sqrt{a^2+(b-z)^2}}\right)$

$$d\vec{E} = 2\pi K \sigma \left(1 - \frac{(b-z)}{\sqrt{R^2+(b-z)^2}}\right)\hat{j}$$

$$\rho = \frac{q}{dV} = \frac{q}{\pi R^2 \, dz} = \frac{\sigma}{dz}$$

$$\sigma = \rho \, dz$$

$$\text{So } dE = 2\pi K \rho \left[1 - \frac{(b-z)}{\sqrt{R^2+(b-z)^2}}\right] dz$$

$$\text{So } E = \int_0^{\ell} 2\pi K \rho \left[1 - \frac{(b-z)}{\sqrt{R^2+(b-z)^2}}\right] dz$$

$$= 2\pi K \rho \int_0^{\ell} 1 - \frac{(b-z)}{\sqrt{R^2+(b-z)^2}} \, dz$$

$$= 2\pi K \rho \left[z\right]_0^{\ell} - 2\pi K \rho \int_0^{\ell} \frac{(b-z)}{\sqrt{R^2+(b-z)^2}} \, dz$$

$$= 2\pi K \rho \ell - 2\pi K \rho \int_0^{\ell} \frac{(b-z)}{\sqrt{R^2+(b-z)^2}} \, dz$$

let $b - z = R\tan\theta$ why???

So $z = b - R\tan\theta$, $dz = -R\sec^2\theta \, d\theta$

$$S_1 \; E = 2\pi K P l + 2\pi K P \int \frac{R \tan\theta \, R \sec^2\theta \, d\theta}{\sqrt{R^2 \sec^2\theta}}$$

$$= 2\pi K P l + 2\pi K P \int R \tan\theta \sec\theta \, d\theta$$

$$= 2\pi K P l + 2\pi K P R \int \frac{\sin\theta}{\cos^2\theta} \, d\theta$$

$$= 2\pi K P l + 2\pi K P R \left[\frac{1}{\cos\theta} \right]$$

$$= 2\pi K P l + 2\pi K P R \left[\frac{1}{\frac{R}{\sqrt{R^2 + (b-z)^2}}} \right]_0^l$$

$$= 2\pi K P l + 2\pi K P R \left[\frac{\sqrt{R^2 + (b-z)^2}}{R} \right]_0^l$$

$$= 2\pi K P l + 2\pi K P \cancel{R} \left[\frac{\sqrt{R^2 + (b-l)^2}}{\cancel{R}} - \frac{\sqrt{R^2 + b^2}}{\cancel{R}} \right]$$

$$= 2\pi K P l + 2\pi K P \left[\sqrt{R^2 + (b-l)^2} - \sqrt{R^2 + b^2} \right]$$

Magnitude

$$E = 2\pi K P l + 2\pi K P \left[\sqrt{R^2 + (b-l)^2} - \sqrt{R^2 + b^2} \right]$$