NDU | **PHS 212 - Electricity & Magnetism**
Faculty of Natural and Applied Sciences | **Midterm Exam I - Spring 2005**
Department of Sciences | **Duration: 100 minutes (1h40)**

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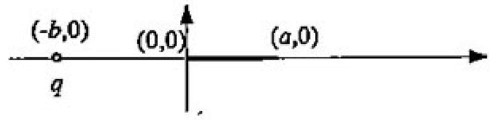
1. Two point charges sum to $-5\mu C$ C. At a separation of 2 cm, they exert a force of 80 N on each other. Find the two charges when
   a. The forces are attractive
   b. The forces are repulsive

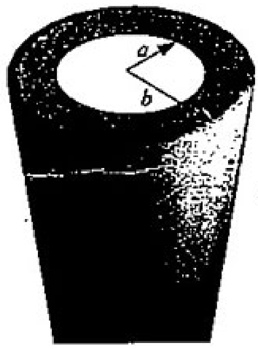
2. A rod of length a whose ends are at (0,0) and (a,0) has a charge density $\lambda=(\frac{Q_0}{a^2})x$ (see figure)

   a. Find the total charge Q on the rod.
   b. Find the force on a charge q at ($-b$,0).
   c. Verify that the force has the correct limit as $b\to\infty$

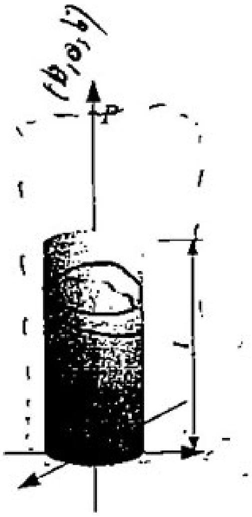


$K\frac{q Q_0}{a^2}\int \frac{x\,dx}{(x+b)^2}$.

3. An infinetly long cylindrical shell of inner radius a and outer radius b carries a uniform volume charge density ρ. Using Gauss' Law, find the electric field for $r<a$, $a<r<b$, and $r>b$.



4. A cylinder, length L and radius R, carries a uniform charge density ρ. Its base is in the z=0 plane (see figure). Find the electric field (magnitude and direction) at the point P on the z-axis (the axis of the cylinder).



$\frac{KqQ_0}{a^2}\int \frac{1}{(x+b)} - \frac{b}{(x+b)^2}$.

$\frac{KqQ_0}{a^2}\left[\ln(x+b)+\frac{b}{x+b}\right]_0^L$ $\quad dV = r\,d\theta\,dr\,dh$

$\frac{KqQ_0}{a^2}\left[\ln(L+b)+\frac{b}{L+b} - \ln b - 1\right]$. $\frac{KqQ_0}{a^2}\left[\frac{L}{L+b}\right)$

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Exam 1

Grade : 88/100

Name :

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Ex 6

N°1:

$Q_1 + Q_2 = -5\,\mu C = -5\cdot 10^{-6}\,C$

$r = 2\,cm = 2\cdot 10^{-2}\,m$

$|F| = 80$

a) If the two charges are attractive so $Q_1 > 0$ and $Q_2 < 0$

$F = \frac{K Q_1 Q_2}{r^2} = 80$

$80 = \frac{-K Q_1(-5\cdot 10^{-6} - Q_1)}{(2\cdot 10^{-2})^2}$

$0.032 = -5\cdot 10^{-6} K Q_1 + K Q_1^2$

$8.9875\cdot 10^{9} Q_1^2 - 44937.5\,Q_1 - 0.032 = 0$

$Q_2 = -5.465\cdot 10^{-6}\,C$

$\cancel{Q_1 = 4.65\cdot 10^{-7}}$ $Q_1 = 6.52\cdot 10^{-7}\,C$

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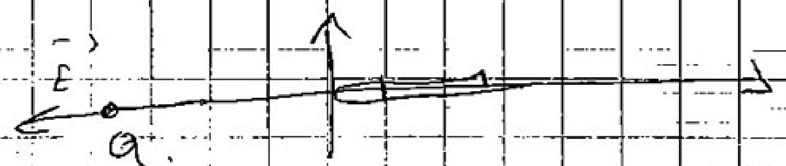
please improve your handwriting

b) If the force are repulsive so Q_1 and $Q_2 > 0$ or Q_1 and $Q_2 < 0$

only the possibility Q_1 and $Q_2 < 0$ is acceptable since b)



$$dE = \frac{K\,dQ\,q}{r^2} = \frac{Kq\lambda\,dx}{(b+x)^2}$$

$$F = \int_0^a \frac{KqQ\,x\,dx}{a^2(b+x)^2} = K\lambda q\int \frac{-dx}{(b+x)^2}$$

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$$= K\lambda q\left[\frac{1}{b+x}\right]_0^a = K\lambda q\left[\frac{1}{b+a} - \frac{1}{b}\right]$$

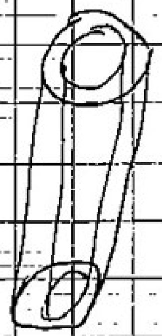
$\cancel{\vec{E} = q\lambda}$ $\cancel{F = K\lambda q\left(\frac{1}{b+a} - \frac{1}{b}\right)}$

c) If $b \to \infty$

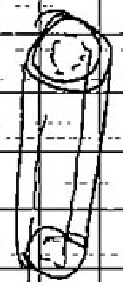
$$\cancel{F = K\lambda q\left[\frac{1}{a}\right] = \frac{kQq}{aa} = \frac{KQq}{a^2}}$$

~~So the ord will ~~be~~ behave like a particle charge point~~

3.)



I $r < a$



Gaussian Surface
is a cylinder
with radius $r < a$
concentric with this shell
Gauss law: The field is radial and is
the same on the Gaussian Surface.

8

The flux in the upper and down surface
is 0 since $\vec{E} \perp \vec{n}$

Gauss law

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

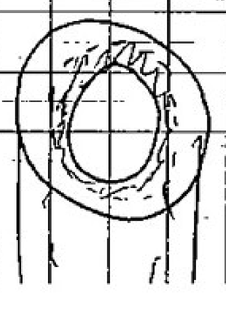
if $r < a$, q_{enc} is 0 so $\Phi = 0$

since $\vec{E}$ and $d\vec{A}$ same direction

$$\Phi = E \oint dA = EA = 0 \text{ So } E = 0$$

If ~~a~~ $a < r < b$:

let ~~as~~ Gaussian Surface is the cylinder concentric with the shell and ~~...~~ with $a < r < b$.



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Name : Georges Barter I.D. N° :

Course N° : Section : Date :

$\phi = \int E\, dA = \frac{Q_{enc}}{\varepsilon_0}$

$Q_{encl} = \rho V$

$V = \pi r^2 l - \pi a^2 l = \pi l (r^2 - a^2)$

~~$\phi = E \oint dA = E(\pi r^2 - \pi a^2)$~~

If ~~a~~ > b

(9) ✓ So $\phi = E \oint dA = E(2\pi r l) = \frac{Q_{en}}{\varepsilon_0}$

$E = \frac{Q_{en}}{2\pi r l\, \varepsilon_0} = \frac{\rho \cancel{\pi} l (r^2 - a^2)}{2\pi r l\, \varepsilon_0}$

$= \frac{\rho (r^2 - a^2)}{2 \varepsilon_0 r} = \frac{1}{2} \frac{\rho (r^2 - a^2)}{r \varepsilon_0}$

If $a > a$ Same principle with Gaussian surface a cylinder with radius > b

$\phi = \oint E \cdot dA = \frac{Q_{enc}}{\varepsilon_0}$

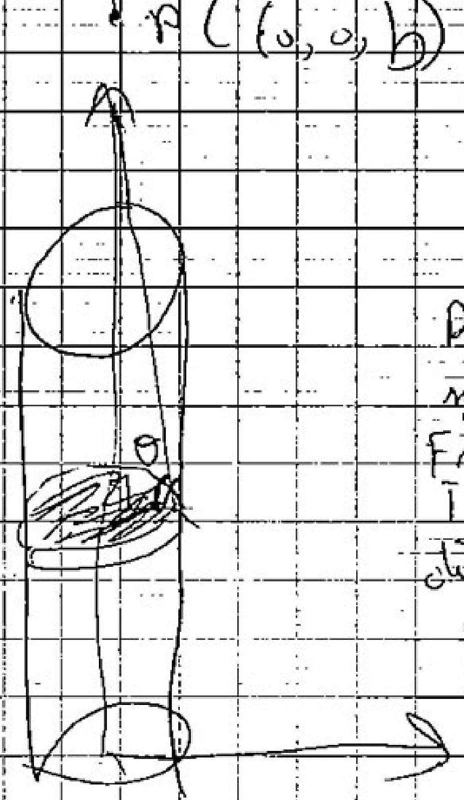
$Q_{enc} = \rho V = \rho(\pi b^2 l - \pi a^2 l)$

$\phi = E\, 2\pi r l$

8 ✓ So $E * 2\pi r l = \frac{\rho(\pi b^2 l - \pi a^2 l)}{\varepsilon_0}$

$$E = \frac{\rho\,(b^2 - a^2)}{2 r \varepsilon_0}$$

Problem 4:



Due to symmetry the field will take the direction of y axis.

$$d\vec{E} = \frac{k\,dq}{z^2}\,\vec{j}$$

let us take the disk of volume:

$dV = \pi R^2\, dz$ as a differential element.

So we know from page 120/

$$d\vec{E} = 2\pi k \sigma\,(b-z)\left(\frac{1}{(b-z)} - \frac{1}{\sqrt{a^2 + (b-z)^2}}\right)$$

$$d\vec{E} = 2\pi K \sigma \left(1 - \frac{(b-z)}{\sqrt{R^2+(b-z)^2}}\right) \hat{j}$$

$$\rho = \frac{q}{dV} = \frac{q}{\pi R^2 \, dz} = \frac{\sigma}{dz}.$$

$$\sigma = \rho \, dz$$

So $dE = 2\pi K \rho \left[1 - \frac{(b-z)}{\sqrt{R^2+(b-z)^2}}\right] dz$

So $E = \int_0^{l} 2\pi K \rho \left[1 - \frac{(b-z)}{\sqrt{R^2+(b-z)^2}}\right] dz.$

$$= 2\pi K \rho \int_0^{l} 1 - \frac{(b-z)}{\sqrt{R^2+(b-z)^2}} \, dz$$

$$= 2\pi K \rho [z]_0^l - 2\pi K \rho \int_0^{l} \frac{(b-z)}{\sqrt{R^2+(b-z)^2}} \, dz$$

$$= 2\pi K \rho l - 2\pi K \rho \int_0^{l} \frac{(b-z)}{\sqrt{R^2+(b-z)^2}} \, dz.$$

let $b - z = R \tan\theta$ why???

So $z = b - R\tan\theta$, $dz = -R\sec^2\theta \, d\theta$

So $E = 2\pi K P l + 2\pi K P \int \frac{R\tan\theta \, R\sec^2\theta \, d\theta}{\sqrt{R^2\sec^2\theta}}$

$= 2\pi K P l + 2\pi K P \int R\tan\theta \sec\theta \, d\theta$

$= 2\pi K P l + 2\pi K P R \int \frac{\sin\theta}{\cos^2\theta} \, d\theta$

$= 2\pi K P l + 2\pi K P R \left[\frac{1}{\cos\theta} \right]$

$= 2\pi K P l + 2\pi K P R \left[\frac{1}{\frac{R}{\sqrt{R^2+(b-z)^2}}} \right]_0^l$

$= 2\pi K P l + 2\pi K P R \left[\frac{\sqrt{R^2+(b-z)^2}}{R} \right]_0^l$

$= 2\pi K P l + 2\pi K P R \left[\frac{\sqrt{R^2+(b-l)^2}}{\cancel{R}} - \frac{\sqrt{R^2+b^2}}{\cancel{R}} \right]$

$= 2\pi K P l + 2\pi K P \left[\sqrt{R^2+(b-l)^2} - \sqrt{R^2+b^2} \right]$

Magnitude

$E = 2\pi K P l + 2\pi K P \left[\sqrt{R^2+(b-l)^2} - \sqrt{R^2+b^2} \right.$

There min is CO

$$\delta O = \frac{K|q_1||q_2|}{r^2}$$

$$0.032 = \frac{5.10^{-6} K q_1 - K q_1^2}{}$$

$$0.032 = 44937.5 q_1 - 8.9875 \times 10^{9} q_1^2$$

$$-8.9875 \times 10^{9} q_1^2 + 44937.5 q_1 - 0.032 = 0$$

$$\delta O = \frac{K |q_1| (5.10^{-6} - |q_1|)}{(2 \cdot 10^{-2})^2}$$

$$K q_1^2 \, 5.10^{-6} - K q_1 = 0.032$$

$$+8.9875 \times 10^{9} q_1^2 - 44937.5 q_1 + 0.032 = 0$$

$q_1 = 8.6 \cdot 10^{-7} C$ $\qquad$ $q_2 = 4.139 \cdot 10^{-6} C$

Problem 2.

$$Q = \int_0^Q dQ = \int_0^a \lambda \, dx$$

$$= \int_0^a \frac{(Q_0)x}{a^2} dx = \frac{Q_0}{a^2}\left[\frac{x^2}{2}\right]_0^a = \frac{a^2 Q_0}{2a^2}$$

$$= \frac{Q_0}{2}$$