Notre Dame University
Faculty of Natural and Applied Sciences
Department of Sciences
Spring 2009-2010

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PHS 203
Exam I

NAME: ___________________________________ **ID: __________________**

SECTION: ___

Only calculators are allowed

Not allowed: mobile phones, any written material, borrowing calculators or pens during the exam.

Write in detail the solutions of the exercises in Part I. Failing to do so will deny you any credit, even if the selected solution happens to be correct. Write in detail the solutions of the problems in Part II.

GRADING

Part I: 10×6 = 60 marks	
Part II Problem 1 :	**20 marks**
Part II Problem 2:	**20 marks**
Total :	**100 marks**

PART I:

1. A 7.00-kg mass is hung from the bottom end of a vertical spring fastened to a ceiling. The mass is set into vertical oscillations with a period of 2.60 s. The spring constant is:
   - **a.** 40.88 N/m
   - **b.** 41 kg/m
   - **c.** 16.9 N/m
   - **d.** 16.91 kg/m
   - **e.** None of the above, my answer is ______________

Solution: $\omega^2 = \dfrac{k}{m} \Rightarrow k = m\omega^2 = m\left(\dfrac{2\pi}{T}\right)^2 = 7\left(\dfrac{2\pi}{2.6}\right)^2 = 40.88\ \text{N/m}$

2. A mass–spring system oscillates with an amplitude of 3.50 cm. If the spring constant is 1250 N/m and the mass is 0.500 kg. The maximum speed of the mass is:
   - **a.** ±1.75 m/s
   - **b.** – 1.75 m/s
   - **c.** 1.75 m/s
   - **d.** 1.75 rad/s
   - **e.** None of the above, my answer is ______________

Solution: $v_m = \pm\omega x_m = \pm\sqrt{\dfrac{k}{m}}x_m = \pm\sqrt{\dfrac{1250}{0.5}}3.5\times10^{-2} = \pm1.75\ \text{m/s}$

3. A string along which waves can travel is 2.70 m long and has a mass of 0.270 kg. The tension in the string is 3.60 N. The frequency of traveling waves of amplitude 7.00×10^{-3} m is 200 Hz. The total mechanical energy for this motion is:
   - **a.** 116 mJ
   - **b.** 116 J
   - **c.** 58 mJ
   - **d.** 58 J
   - **e.** None of the above, my answer is______________

Solution: $E = \mathscr{P}\times T = \dfrac{1}{2}\mu\omega^2 y_m^2 vT = \dfrac{1}{2}\left(\dfrac{m}{L}\right)(2\pi f)^2\ y_m^2\sqrt{\dfrac{\tau}{\mu}}\left(\dfrac{1}{f}\right) = 116\ \text{mJ}$

4. In an engine, a piston oscillates with simple harmonic motion so that its displacement varies according to the expression $x = (5.00\ \text{cm})\cos\left(2t + \pi/6\right)$ where x is in centimeters and t is in seconds. At t = 0 s, the period of the motion is:

a. 3.14 s
b. $\pi/6$ s
c. 6.2 s
d. 0 s
e. None of the above, my answer is _____________

Solution: $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{2} = \pi \text{ s} \approx 3.14 \text{ s}$

5. Within the study of damped oscillations, simple harmonic motion occurs in the case of:
   a. critical damping in which the displacement follows $x = e^{-\gamma t}\left(A + Bt\right)$
   b. underdamping in which the displacement follows $x = Ce^{-\gamma t}\cos\left(\lambda t + \varphi\right)$
   c. overdamping in which the displacement follows $x = e^{-\gamma t}\left(Ae^{\alpha t} + Be^{-\alpha t}\right)$
   d. all of the above three
   e. answers **a** and **b**

Here A, B, and C are constants, $\gamma = \dfrac{\beta}{2m}$ (where β is the damping coefficient), $\lambda = \sqrt{\omega^2 - \gamma^2}$ (where ω is the angular frequency of oscillations) and $\alpha = \sqrt{\gamma^2 - \omega^2}$.

Solution: Only the underdamping case can be considered as simple harmonic motion due to its sinusoidal character brought by the cosine function.

6. A physical pendulum moves in simple harmonic motion with a frequency of 450 Hz. This pendulum has a mass of 2.20 kg and the pivot is located 0.350 m from the center of mass. Its moment of inertia is:
   a. 9.44×10^{-7} kg.m^2
   b. 9.44×10^{-7} N.m^2
   c. 59.31×10^{-7} kg.m^2
   d. 59.31×10^{-7} N.m^2
   e. None of the above, my answer is _____________

Solution: $T = \dfrac{1}{f} = 2\pi\sqrt{\dfrac{I}{mgd}} \Rightarrow I = \dfrac{mgd}{4\pi^2 f} = 9.44\times10^{-7} \text{ kg.m}^2$

7. Ocean waves with a crest-to-crest distance of 10.0 m can be described by the equation $y\left(x,t\right) = 0.8\sin\left[0.628\left(x - vt\right)\right]$ where $v = 1.20$ m/s . The wavelength is:

a. 10 m
b. 1.6 m
c. 0.8 m
d. 3.946 m
e. None of the above, my answer is ______________

Solution: The separation between two consecutive crests is the wavelength, hence $\lambda = 10\text{ cm}$.

8. A phone cord is 4.00 m long. The cord has a mass of 0.200 kg. A transverse wave pulse is produced by plucking one end of the taut cord. The pulse makes four trips down and back along the cord in 0.800 s. The tension in the cord is:
   a. 20 N
   b. 5 N
   c. 80 N
   d. 500 N
   e. None of the above, my answer is______________

Solution: $v = \sqrt{\dfrac{\tau}{\mu}} \Rightarrow \tau = v^2\mu = \left(\dfrac{L}{t/4}\right)^2\left(\dfrac{m}{L}\right) = \left(\dfrac{4}{0.8/4}\right)^2\left(\dfrac{0.2}{4}\right) = 20\text{ N}$

9. A wave on a string is described by the wave function $y = 0.1\sin\left(0.50x - 20t\right)$.The frequency of oscillation at $x = 2.00$ m is:
   a. 3.183 Hz
   b. 125.66 Hz
   c. 20 Hz
   d. 1 Hz
   e. None of the above, my answer is ______________

Solution: $f = \dfrac{\omega}{2\pi} = \dfrac{20}{2\pi} \approx 3.183\text{ Hz}$

10. Transverse pulses travel with a speed of 200 m/s along a taut copper wire whose diameter is 1.50 mm. The density of copper is 8.92 g/cm^3. The tension in the wire is:
   a. 630 N
   b. 63 N
   c. 3.941 N
   d. 40 N
   e. None of the above, my answer is ______________

Solution: $v = \sqrt{\dfrac{\tau}{\mu}} \Rightarrow \tau = v^2\mu = v^2\dfrac{m}{L} = v^2\dfrac{\rho_{Cu}V}{L} = v^2\rho_{Cu}\dfrac{\pi D^2}{4} \approx 630\text{ N}$

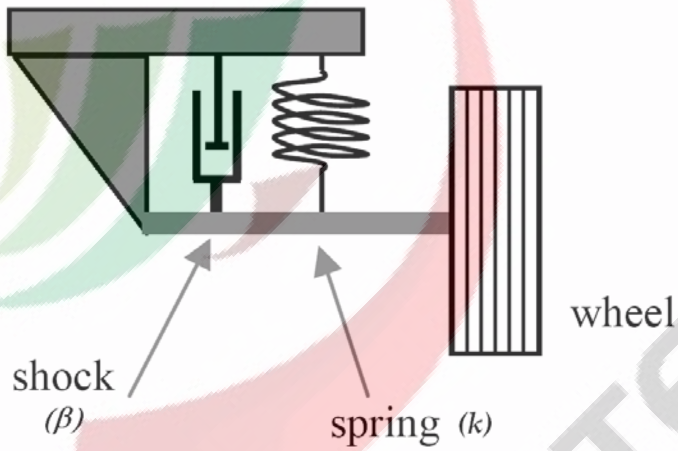
PART II

Problem 1

A simple pendulum 20 cm long, hanging from the ceiling of an automobile, is oscillating in a simple harmonic motion.

a. What is the frequency of this pendulum if the automobile is still with respect to the ground?

b. When the tire of this moving car hits a bump, the car's suspension system brings it back to equilibrium reasonably quickly (see the figure below). In practice the damping coefficient $\beta = 2\times10^3$ kg/s and the spring constant $k = 5\times10^4$ N/m are adjusted so that the tire is slightly underdamped. It is important to mention that the car's mass of 2400 kg is equally supported by the four wheels. What is the angular frequency of the oscillation on one wheel?



Solution:

a. The frequency for small amplitude oscillations is $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, where L is the length of the pendulum. This gives $f = \frac{1}{2\pi}\sqrt{\frac{9.8}{0.20}} = 1.114$ Hz.

b. The angular frequency of the damped oscillator on one wheel is $\omega' = \sqrt{\frac{k}{m} - \frac{\beta^2}{4m^2}}$ where m is the mass supported by a wheel. Thus $\omega' = 8.975$ rad/s.

Problem 2

An aluminum wire, of length L_1 = 60.0 cm, cross-sectional area 757.73 cm^2, and density 2.60 g/cm^3, is joined to a steel wire, of density 7.80 g/cm^3 and the same cross-sectional area. The compound wire is set to hang vertically from a ceiling. A block of mass $m = 10$ kg is attached to the lower end of this compound so that the distance L_2 from the joint to the lower end is 86.6 cm. Transverse waves are set up on the wire by an external source of variable frequency; a node is located at the lower end. Another node is located at the joint.

- **a.** Find the lowest frequency that generates a wave.
- **b.** How many antinodes are observed at this frequency?

Solution:

- **a.** The frequency f of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are n_1 loops in the aluminum section of the wire. Then, $L_1 = n_1 \frac{\lambda_1}{2} = n_1 \frac{v_1}{2f}$, where λ_1 is the wavelength and v_1 is the wave speed in that section. Thus we obtain $f = n_1 \frac{v_1}{2L_1}$. A similar expression holds for the steel section: $f = n_2 \frac{v_2}{2L_2}$. Since the frequency is the same for the two sections, we obtain $n_1 \frac{v_1}{2L_1} = n_2 \frac{v_2}{2L_2}$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\frac{\tau_1}{\mu_1}}$, where μ_1 is the linear density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1 V_1 = \rho_1 A L_1$, where ρ_1 is the density (mass per unit volume) for aluminum and A is the cross-sectional area of the wire. Thus $\mu_1 = \frac{\rho_1 A L_1}{L_1} = \rho_1 A$ and $v_1 = \sqrt{\frac{\tau_1}{\rho_1 A}}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\frac{\tau_2}{\rho_2 A}}$. We note that the cross-sectional area is the same for the two sections. The tension for the steel section is $\tau_2 = mg$. On the other hand, the tension for the aluminum section is $\tau_1 = mg + m_2 g = mg + \left(\rho_2 A L_2\right) g$, where m_2 is the mass for the steel section. The equality of the frequencies for the two sections now leads to $\frac{n_2}{n_1} = \frac{L_2}{L_1}\sqrt{\frac{\tau_1 \rho_2}{\tau_2 \rho_1}} = 2$. The smallest integers that have the latter ratio are n_1=1 and n_2 = 2. The frequency is $f = 0.470 \text{ Hz} = 470 \text{ mHz}$.
- **b.** The standing wave pattern has one loop in the aluminum section and 2 loops in the steel section. This results in a total of 3 loops, *i.e.* 6 antinodes.