Math 201 - Exam 1 (Fall 10)

T. Tlas

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- Please answer question 3 on the same sheet of paper on which it is written. Question 1 has an extra sheet and question 2 has two extra sheets for you to write your answer on them. Any part of your answer written on the wrong page will not be graded.
- When finished leave your work on your desk for it to be collected by the proctors.
- There are 3 problems in total. All questions have several parts to them. Make sure that you attempt them all.
- This is a closed book exam and no calculators are allowed.

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Name :

ID # :

Section :

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$Q1$	
$Q2$	
$Q3$	
$TOTAL$	

Problem 1 (10 points each) Which of the following series converge and which diverge? Those which converge, do they converge absolutely or conditionally? When possible find the sum of the convergent series. Provide brief justifications for your answers.

i-

$$\sum_{n=1}^{\infty} \left(1 - \frac{2}{n^2}\right)^{n^2}$$

ii-

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^n}{\pi^{2n}}$$

iii-

$$\sum_{n=1}^{\infty} n^2 e^{-n}$$

iv-

$$1 + \frac{\frac{1}{2}}{1!}\left(\frac{1}{2}\right) + \frac{(\frac{1}{2})(\frac{-1}{2})}{2!}\left(\frac{1}{2}\right)^2 + \frac{(\frac{1}{2})(\frac{-1}{2})(\frac{-3}{2})}{3!}\left(\frac{1}{2}\right)^3 + \frac{(\frac{1}{2})(\frac{-1}{2})(\frac{-3}{2})(\frac{-5}{2})}{4!}\left(\frac{1}{2}\right)^4 + \dots$$

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ADDITIONAL SHEET FOR PROBLEM 1 ANSWER

Problem 2 Answer (with brief justification) the following questions:

i- (10 points) Find the Taylor series (centred at 0) of

$$\sin(x^2) - x^2 + \frac{x^6}{6}.$$

Write it in the form $\sum_{n=0}^{\infty} c_n x^n$ (i.e. find the coefficient c_n as a function of n, where n starts at zero). What is the radius of convergence of this series?

ii- (10 points) Find the following limit

$$\lim_{x\to 0} \frac{\sin(x^2) - x^2 + \frac{x^6}{6}}{x^{10}}.$$

iii- (5 points) Is $\sin(\frac{1}{100})$ bigger or smaller than $\frac{1}{100} - \frac{1}{6000000}$?

iv- (10 points) Does the series

$$\sum_{n=1}^{\infty} \left(\sin\Big(\frac{1}{n^2}\Big) - \frac{1}{n^2} + \frac{1}{6n^6} \right)$$

converge or diverge?

v- (10 points) Does the series

$$\sum_{n=1}^{\infty} \left(\sin\Big(\frac{1}{n^2}\Big) - \frac{1}{n^2} + \frac{1}{6n^6} \right) n^8$$

converge or diverge?

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ADDITIONAL SHEET FOR PROBLEM 2 ANSWER

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Problem 3

i- (9 points) Using the Taylor series for the sine find the following definite integral as a series

$$\int_0^{0.1} \frac{\sin(x)}{x}\, dx.$$

ii- (6 points) Estimate the above integral with an error whose absolute value is less than $\frac{1}{10^6}$. Is your answer an over- or an under-estimate?

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