92

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Part II. Solve the following problems

(15 points)

1. A damped harmonic oscillator is described by the equation $\ddot{x}+20\dot{x}+1600x=0$ It starts its motion at t = 0 from $x_0=10m$ with an initial speed $\dot{x}_o=-5m/s$.

a. Write down the solution $x(t)$ that obeys the given initial conditions.

b. Determine the different features of the displacement solution, i.e. the time constant, the period if it exists.

a) ~~$x(t) = X_m \cos(\omega t + 2\phi)$~~ the equation of a damped harmonic oscillator is given by

$$X(t) = X_m e^{-bt/2m} \cos(\omega' t + \phi)$$

At $t=0$, $x_0 = 10m$

$$x(0) = 10 = X_m \cos\phi$$

$$v(t) = X_m \left[\frac{-b}{2m} e^{-bt/2m} \cos(\omega' t + \phi) - \omega' e^{-\frac{bt}{2m}} \sin(\omega' t + \phi) \right]$$

$$v(0) = X_m \left[\frac{-b}{2m} \cos\phi - \omega' \sin\phi \right] = -5$$

* $-bv - kx = mx''$

$-bx' - kx = mx''$

$$x'' + \frac{b}{m}x' + \frac{k}{m}x = 0$$

$$\frac{b}{m} = 20, \quad \frac{k}{m} = 1600$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{1600 - \frac{1}{4}(20)^2} = 38.73 \text{ rad/s}$$

$$v(0) = -X_m \left[10\cos\phi + 38.73 \sin\phi \right] = -5$$

$$X(0) = 10 = X_m \cos\phi$$

$$\frac{v(0)}{X(0)} = \frac{-5}{10} = \frac{-X_m\left[10\cos\phi + 38.73\sin\phi\right]}{X_m \cos\phi}$$

$$0.5\cos\phi = 10\cos\phi + 38.73\sin\phi$$

$$-9.5\cos\phi = 38.73\sin\phi$$

$$\tan\phi = \frac{-9.5}{38.73} = -0.245$$

15

$$\phi = -13.78^\circ$$

$$= 180^\circ - 13.78^\circ$$

$$= 166.2^\circ$$

$$2\pi \rightarrow 360^\circ$$

$$x \rightarrow 166.2^\circ$$

$$x = \phi = 2.9\text{ rad}$$

$$x(0) = 10 = X_m \cos(166.2) \Rightarrow X_m = |-10.29| \text{ (always +ve)}$$

$$= 10.29\text{ m} \simeq 10.3\text{ m}$$

$$X(t) = 10.3\, e^{-10t} \cos(38.73t + 2.9)$$

(m) (s) (sec)

b) ~~Pe~~ Period: $\omega' = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega'} = 0.162\text{ sec}$

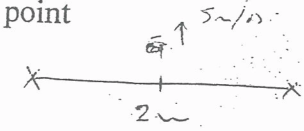
(25 points)

3. A horizontal rope of mass 1.2 kg is fixed at both ends ($x = 0$ and $x = 2$ m). It is made to oscillate up and down at its fundamental mode, at frequency 5 Hz. At $t = 0$, the point $x = 1$ m has zero displacement and is moving upward in the positive direction of a y axis with a transverse velocity of 5 m/s.

a. What is the tension in the rope?

b. Write the standing wave function for the fundamental mode obeying the initial conditions of the problem. What is the phase constant of the wave function?

c. Determine the amplitude of the motion of that point



$m = 1.2\,Kg$

$f = 5\,hz$

a) The speed of the wave in the string is given by.

6

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow v^2 \mu = T = (25)\left(\frac{1.2}{2}\right) = 15\,N$$

$$\mu = \frac{m}{L} = \frac{1.2\,Kg}{2\,m}$$

$\mu = 0.6\,kg/m.$

b) $y = 2y_m \sin(kx) \cos(\omega t)$

2

$$y(x,t) = 2y_m \sin(kx)\cos(\omega t)$$

$$\omega = 2\pi f = 2\pi \times 5 = 10\pi \text{ rad/s}$$

2

$k = \frac{2\pi}{\lambda}$ and ~~$\lambda = vf = 5 \times 5 = 25\,m$~~.

At fundamental Mode: $(n = 1)$

1

$$f = \frac{v}{2L} \Rightarrow v = 5 \times 2 \times 2 = 20\,m/s$$

$$f = \frac{v}{\lambda} \Rightarrow \lambda = \frac{v}{f} = \frac{20}{5} = 4\,m$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4} = 0.5\pi\ m$$

2

$$y(x,t) = 2y_m \sin(0.5\pi\, x)\cos(10\pi\, t)$$

x in m, t in sec

5

Phase constant is $= \phi = \frac{3\pi}{2}$ rad ⟺ prove

since :

we Replace $x = 1m$ and $t = 0$;

$$y = 2y_m \sin(0.5\pi)\cos(0+\phi)$$

$$\boxed{0 = 2y_m \sin(0.5\pi)\cos\phi}$$

$$u = \frac{\partial y}{\partial t} = -10\pi\, 2y_m \sin(0.5\pi x)\sin(10\pi t)$$ ✓

$$= -20\pi\, y_m \sin(0.5\pi x)\sin(10\pi t)$$ ✓

$$u(1;0) = -20\pi y_m \sin(0.5\pi)\sin(0+\phi) = 5$$ ✓

$$u = -20\pi y_m \sin\phi = 5.$$ ✓

$$0 = 2y_m \sin(0.5\pi)\cos\phi$$

then $\phi = \frac{3\pi}{2}$ rad. for part (b).

$\cancel{\phi = 2y_m \sin(0.5\pi)\cos}$

c) $\frac{5}{-20\pi \sin\frac{3\pi}{2}} = y_m = 0.079m.$

(6)