PART I

1. The period of a pendulum is 5 s at a point where $g = 9.81\ m/s^2$. If the pendulum were on the moon, where the acceleration due to gravity is one-sixth that on earth, its period is:

   a. 12.2 s
   b. 30 s
   c. 5/6 s
   d. 5 s
   e. None of the above, my answer is ~~900s~~ 150s.

2. A level platform vibrates horizontally with simple harmonic motion with a period of 0.8 s. A box on the platform starts to slide when the amplitude of vibration reaches 10 cm. The coefficient of static friction between the body and the platform is:

   a. 0.52
   b. 0.63
   c. 0.78
   d. 0.45
   e. None of the above, my answer is________

3. A taut string for which $\mu = 5.00\times10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

   a. 512 W
   b. 256 W
   c. 5210 W
   d. 1024 W
   e. None of the above, my answer is 206.06Watts.

4. Two identical waves travel in opposite directions along a string with a speed of 20 cm/s. If the time interval between instants when the string is flat is 0.25s, the wavelength of the waves is:

   a. 5.0 cm
   b. 10.0 m
   c. 20.0 cm
   d. 10.0 cm
   e. None of the above, my answer is ________

5. The amplitudes of two sinusoidal string waves are 3.0 cm and 4.0 cm, and they have the phase constants of 0 and $\pi/2$ rad, respectively. They have the same frequency and they are traveling in the same direction and medium. When they are combined, the amplitude of the resultant wave is:

   a. 1.0 cm
   b. 3.5 cm
   c. 7.0 cm
   (d.) 5.0 cm
   e. None of the above, my answer is______________

6 $y' = \sqrt{(3)^2+(4)^2} = 5$ cm

6. A particle has a displacement $x = 0.4\cos(3t + \pi/4)$, where x is in meters and t is in seconds. At what time after $t = 0$ does the maximum velocity first occur?

   (a.) 1.31 s
   b. 3.1 s
   c. 13.1 s
   d. 0.26 s
   (e.) None of above, my answer is 0.072s

max $v \Rightarrow \cos 3t + \frac{\pi}{4} = 1$

$3t + \frac{\pi}{4} = 1$

$\Rightarrow t = 0.072$ s

7. A mass spring system oscillates with an amplitude of 3.5 cm. If the spring constant is 250 N/m, the mechanical energy of the system is:

   a. 1.15 J
   (b.) 0.15 J
   c. 1.1 J
   d. 0.25 J
   e. None of the above, my answer is__________

6 $E = \frac{1}{2} k X_m^2 = \frac{1}{2}(250)(0.035)^2 = 0.15$ J

8. Two sinusoidal waves combining in a medium are described by the wave functions $y_1 = (3\text{cm})\sin(\pi x+0.6t)$ and $y_2 = (3\text{cm})\sin(\pi x-0.6t)$, where x is in cm an t in seconds. Antinodes could occur at:

   (a.) 1.5 cm
   b. 1.0 cm
   c. 2.0 cm
   d. 3.0 cm
   e. None of the above.

6 $kx = (n+\frac{1}{2})\pi$

$\frac{2\pi}{\lambda} x = (n+\frac{1}{2})\pi$

$\Rightarrow$

$\pi x = n\frac{\pi}{2}$

$\pi k = (2k+1)\frac{\pi}{2}$

9. The position of a particle is given by x = 2.5 cos π t, where x is in meters and t is in seconds. The acceleration of the particle when x = 1.5 m is

a. 14.8 m/s²
(b.) -14.8 m/s²
c. 6.28 m/s²
d. -6.28 m/s²
e. None of the above.

$a = -\omega^2 X_m = (3.14)^2 (1.5)$
$= -14.8 \, m/s^2$

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10. Two identical waves, moving in the same direction along a stretched string, interfere with each other. The combined wave has an amplitude 1.5 times that of the common amplitude of the two combining waves. The phase difference between the two waves is:

a. 0.732 rad
(b.) $\pm$0.732 rad
c. $\pm$41.4°
d. $\pm$0.23λ
e. None of above, my answer is ____________

$\pm$ 1.445 rad

$y_m = 2 y_m \cos\frac{\phi}{2} = 1.5 y_m$

$y'(x,t) = 2y_m \cos\frac{\phi}{2} \sin(kx - \omega t)$

$1.5 y_m \Rightarrow \cos\frac{\phi}{2} = \frac{1.5}{2} = 0.75$

$\Rightarrow \frac{\theta}{2} = \pm \cos^{-1} 0.75$

$\Rightarrow \theta = \pm 1.45$ rad

PART II

Problem 1

A massless spring hangs from the ceiling. A small object is attached to its lower end. The object is initially held at rest in a position where the spring is unstretched and then released. It oscillates up and down, with its lowest position being 10 cm below the initial position.

a) What is the frequency of the oscillation?

b) What is the speed of the object when it is 8 cm below the initial position?

a) $X_m = \frac{10\ cm}{2} = \frac{0.1 m}{2}$

b) $v = \omega X_m$

$\omega = 2\pi f$

$X_m = 0.08\ cm$

a) $kx = mg$

$\omega^2 m \cdot 0.05 = mg$

$\omega = \sqrt{\frac{9.81}{0.05}} = 14\ rad/s$

$f = \frac{\omega}{2\pi} = 2.228\ Hz$

b) $v \to 8\ cm$

$x = -3$ in $x(t) = 5_{cm} \cos(14t) \Rightarrow t = 0.158\ s$

in $v(t) = -14(0.05)\sin(14 \times 0.158)$

$v = -0.56\ m/s$

$v = \frac{\omega}{k}$

Problem 2 $v = 36$ m/s $f = 60$ Hz $\lambda = \frac{v}{f} = \frac{36}{60}$

A string with both ends held fixed is vibrating in its *second-harmonic* mode. The waves have a speed of 36 m/s and a frequency of 60 Hz. The amplitude of the standing wave at an antinode is 0.6 cm. $k = \frac{2\pi}{\lambda}$

a) Calculate the amplitude of the motion of points on the string a distance of
   i) 30 cm; 0
   ii) 15 cm; and 0.6
   iii) 7.5 cm 0.42

   from the left-hand end of the string.

   $y_1 = y_m \sin(kx - \omega t)$

   $y_2 = y_m \sin(kx + \omega t)$

b) At each of the points in part a), how much time does it take the string to go from its largest upward displacement to its largest downward displacement?

$y' = y_1 + y_2 = 2 y_m \sin kx \cos \omega t$ $= 2 y_m \sin kx \cos \omega t$

a) $v = 36$ m/s $f = 60$ Hz

i) $f = \frac{n v}{2L} = \frac{2(36)}{2(0.3)} = 120$ Hz

ii) $f = \frac{2(36)}{2(0.15)} = 240$ Hz

iii) $f = \frac{2(36)}{2(0.075)} = \frac{72}{0.15} = 480$ Hz

$y'(x,t) = 0.6\,\text{cm} \sin kx \cos \omega t$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.6}$

$\lambda = \frac{v}{f} = \frac{36\ \text{m/s}}{60\ \text{Hz}} = 0.6$ m $= 60$ cm

$y = 0.6\,\text{cm} \sin\left(\frac{2\pi}{0.6} \times 0.3\right)$

$y = 0 \rightarrow$ nod.

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b) i) $T = \frac{1}{f} = \frac{1}{120} = 8.33 \times 10^{-3}$ s.

ii) $T = \frac{1}{240} = 4.166 \times 10^{-3}$ s

iii) $T = \frac{1}{480} = 2.083 \times 10^{-3}$ s