Introduction to Differential Geometry

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Exam 1 — 11 March 2019

Exercise 1. 30 P.

Let $\varphi\colon \mathbb{R}^2 \to \mathbb{R}^3,\ (u,v) \mapsto (u, v, u^3 - 3uv^2)$.

(a) Show that $S = \varphi(\mathbb{R}^2)$ is a surface (which is called *monkey saddle*), and give parametrizations and charts that cover this surface.

(b) For each point $p \in S$, give a basis of the tangent space T_pS.

(c) Let $f\colon S \to \mathbb{R}$ be given by $f(x,y,z) = z$. Find all points $p \in S$ where $d_pf = 0$.

Exercise 2. 30 P.

(a) State the definition of the Frenet frame (T, N, B) for a p.a.l. curve $\gamma\colon (a,b) \to \mathbb{R}^3$ and state the corresponding Frenet–Serret equations. What assumptions on the curve need to be satisfied?

(b) Let $\widehat{\gamma}$ be the curve obtained from projecting γ orthogonally to the plane through the point $\gamma(t_0)$ with normal $B(t_0)$. How is the curvature of $\widehat{\gamma}$ in the point $\gamma(t_0)$ related to the curvature of γ in the point $\gamma(t_0)$?

Exercise 3. 40 P.

Let $\gamma\colon (0, 2\pi) \to \mathbb{R}^3$ be the following curve, called *Viviani's curve.*

$$\gamma(t) = \left(\frac{1}{2} \cdot (1 + \cos(t)), \frac{1}{2}\sin(t), \sin(t/2)\right)$$

Calculate curvature κ and torsion τ of γ.