Math 201 — Fall 2009–10
Calculus and Analytic Geometry III, sections 1–8, 24–26
Quiz 2, December 2 — Duration: 1 hour

GRADES:

1 (/15)	2 (/15)	3 (/15)	4 (/16)	5 (/19)	6 (/20)	TOTAL/100

YOUR NAME:

YOUR AUB ID#:

PLEASE CIRCLE YOUR SECTION:

Section 1 Lecture MWF 3 Professor Makdisi Recitation F 11 Ms. Nassif	Section 2 Lecture MWF 3 Professor Makdisi Recitation F 2 Ms. Nassif	Section 3 Lecture MWF 3 Professor Makdisi Recitation F 4 Ms. Nassif	Section 4 Lecture MWF 3 Professor Makdisi Recitation F 9 Ms. Nassif
Section 5 Lecture MWF 10 Professor Raji Recitation T 11 Professor Raji	Section 6 Lecture MWF 10 Professor Raji Recitation T 3:30 Ms. Itani	Section 7 Lecture MWF 10 Professor Raji Recitation T 8 Ms. Itani	Section 8 Lecture MWF 10 Professor Raji Recitation T 2 Ms. Itani
Section 24 Lecture MWF 2 Professor Tlas Recitation F 11 Dr. Yamani	Section 25 Lecture MWF 2 Professor Tlas Recitation F 12 Dr. Yamani	Section 26 Lecture MWF 2 Professor Tlas Recitation F 3 Professor Tlas	

INSTRUCTIONS:

1. **Write your NAME and AUB ID number, and circle your SECTION above.**
2. **Solve the problems inside the booklet. Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit.**
3. **You may use the back of each page for scratchwork OR for solutions. There are three extra blank sheets at the end, for extra scratchwork or solutions. If you need to continue a solution on another page, INDICATE CLEARLY WHERE THE GRADER SHOULD CONTINUE READING.**
4. **Closed book and notes. NO CALCULATORS ALLOWED. Turn OFF and put away any cell phones.**

GOOD LUCK!

An overview of the exam problems.
Take a minute to look at all the questions, THEN
solve each problem on its corresponding page INSIDE the booklet.

1. Let the function $f(x)$ be given by

$$f(x) = \begin{cases} 0, & \text{when } 0 \leq x < \pi \\ x - \pi, & \text{when } \pi \leq x < 2\pi \\ \text{and } f(x) \text{ is periodic with period } 2\pi. \end{cases}$$

a) (5 pts) Sketch the graph of $f(x)$ for $x \in [-2\pi, 4\pi]$.

b) (10 pts) The Fourier series of $f(x)$ is $\sum_{n\geq 0} a_n \cos nx + \sum_{n\geq 1} b_n \sin nx$. Find ONLY the coefficients b_n.

2. a) (6 pts) Plot the polar graph of the curve $C : r = 1 + \sin\theta$. Also draw the line $L : y = 4/9$ on your graph.

b) (3 pts) Convert the equation of L to polar coordinates.

c) (6 pts) Find the (r, θ)-coordinates of the two points of intersection on $L \cap C$.

3. Consider the following moving point in space:

$$P(t) = (3t, \sqrt{6}\, e^t, \frac{1}{2}e^{2t}).$$

a) (5 pts) Find the velocity and the speed of $P(t)$ at the instant $t = 0$.

b) (5 pts) What is the arclength of the curve given by $P(t)$ for $0 \leq t \leq \ln 5$? Simplify your answer.

c) (5 pts) Suppose we have a function $f(x, y, z)$ with the property

$$\vec{\nabla} f|_{(3,\sqrt{6}\,e,\frac{1}{2}e^2)} = (e^2, -\sqrt{6}\,e, 5).$$

Find $\frac{d}{dt} f(P(t))$ at the instant when the point $P(t)$ passes through $(3, \sqrt{6}\,e, \frac{1}{2}e^2)$.

4. a) (8 pts) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$ does not exist.

b) (8 pts) Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^2 + y^2}$ does exist (hint: the limit is equal to 0).

5. Consider the function $f(x, y, z) = z e^{x^3 y}$.

a) (6 pts) Find the gradient of $f(x, y, z)$ at $P_0(1, 1, -1)$.

b) (7 pts) Find the equation of the tangent plane to the surface $f(x, y, z) = -e$ at P_0.

c) (6 pts) Determine the direction in which $f(x, y, z)$ increases most rapidly when the point (x, y, z) moves away from P_0. Your answer should be a **unit** vector.

6. Given a function $f(x, y)$ satisfying $f(1, 2) = 4$, $\quad \vec{\nabla} f \Big|_{(1,2)} = (3, 4)$.

a) (6 pts) Approximately how much is $f(1.03, 1.99)$?

b) (7 pts) Find a direction $\vec{u}$ in which the directional derivative $D_{\vec{u}} f \Big|_{(1,2)} = 0$. Your answer $\vec{u}$ should be a **unit** vector.

c) (7 pts) Let S be the graph of f. In other words, $S = \{(x, y, z) \in \mathbf{R}^3 \mid z = f(x, y)\}$. Find the equation of the tangent plane to S at the point $P_0(1, 2, 4) \in S$. (Be careful.)

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