Name: __

MEN 330
EXAM 1
FALL 2003

ID! 02 - 0364

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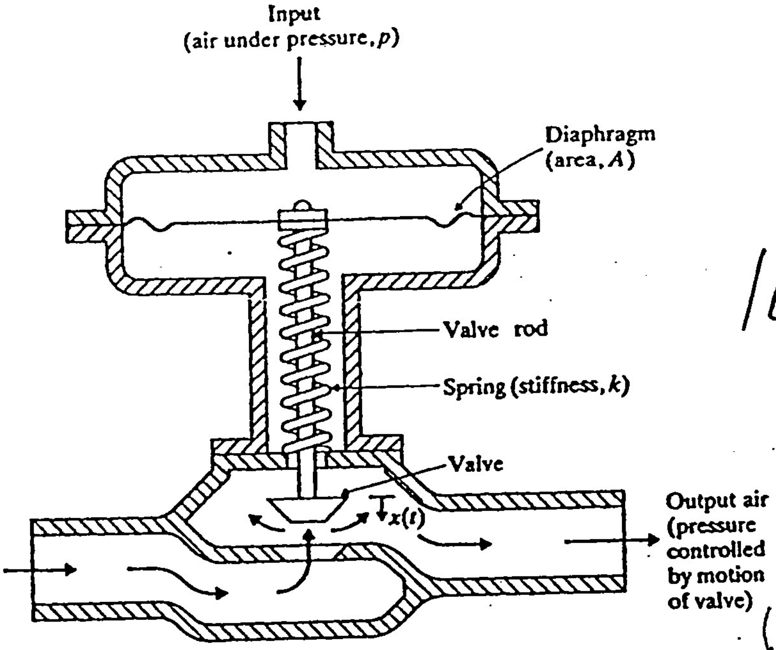
1) (15 pts) The spring actuator shown in the figure operates using the air pressure from a pneumatic controller (p) as input and providing an output displacement to a valve (x) proportional to the input air pressure. The diaphragm, made of a fabric base rubber, has an area A and deflects under the input air pressure against a spring of stiffness k. Find the steady-state response of the valve under a harmonically fluctuating input air pressure $p(t) = p_0 \sin \omega t$ for the following data: $p_0 = 10\,\text{psi}$, $\omega = 8\,\text{rad/s}$, $A = 100\,\text{in}^2$, $k = 400\,\text{lb/in}$, weight of spring $= 15\,\text{lb}$, weight of valve and valve rod $= 20\,\text{lb}$



A spring actuator.

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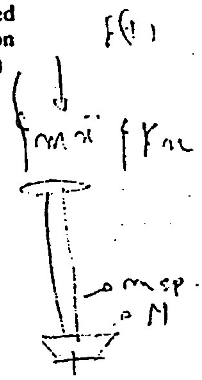


$P_0 = \frac{F_0}{A} \Rightarrow F_0 = P_0 \times A = 10\frac{lb}{in^2} \times 100\,in^2 = 1000\ lb.$

$\Rightarrow F(t) = F_0 \sin \omega t = 1000 \sin 8t.$

Harmonically excited system.

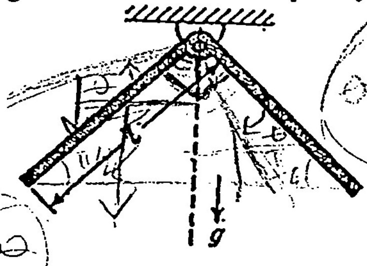
$m_{eq}\ddot{x} + Kx = F(t) = 1000 \sin 8t.$

$\omega_n = \sqrt{\frac{K}{m_{eq}}}$; with $m_{eq} = M + \frac{m_{spr.}}{3} = \frac{20}{32.2\times12} + \frac{15}{32.2\times12\times3} = 0.06469$ X

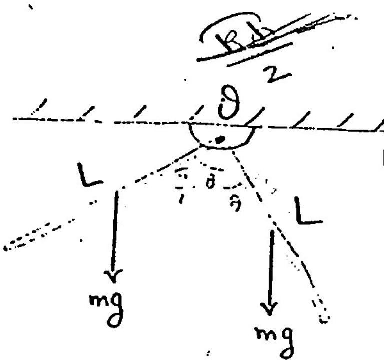
$\omega_n = \sqrt{\frac{400}{0.06469}} = 78.63419$

(steady state

2) (15 pts) A bar in the form of a 90-degree bend has a total mass 2m and a total length 2l (each arm of mass m and length l). It hangs from a pivot in a vertical plane. Gravity is acting. Find the natural frequency for small oscillations.



$\alpha = ?,\ \frac{\pi}{2} -$



$\alpha = \frac{\pi}{2} - \theta$

$(\sin\alpha)\ \frac{l}{2}$

$\sin\alpha = \alpha$

$\alpha = \frac{\pi}{4} + \theta$

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$\sum M_0 = I\ddot{\theta}$ (+) ✓

$\Rightarrow -mg\frac{L}{2}\sin\left(\frac{\pi}{2}-\theta\right) + mg\frac{L}{2}\sin\theta = I\ddot{\theta}$

$I = 2\int r^2 dm = 2\int_0^L r^2 \rho A\, dr = 2\rho A \frac{r^3}{3}\Big|_0^L = \frac{2}{3}\rho A L^3 = \frac{2}{3} m L^2$ ✓

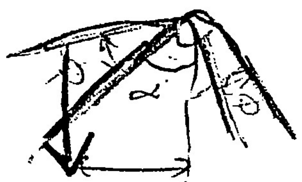
$\Rightarrow \frac{2}{3} m L^2 \ddot{\theta} - mg\frac{L\sin\theta}{2} + mg\frac{L}{2}\cos\theta = 0$

$\frac{\pi}{2} = \alpha + \theta + \frac{\pi}{4}$

for small oscillations $\sin\theta \simeq \theta$ and $\cos\theta \simeq 1$

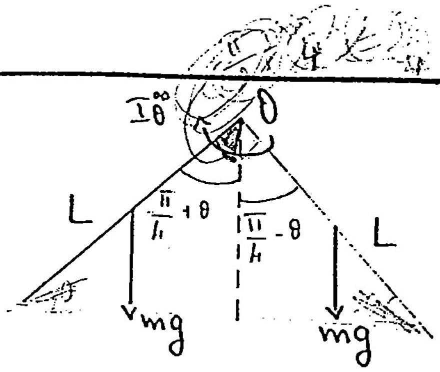
$\Rightarrow \frac{2}{3} m L^2 \ddot{\theta} - mg\frac{L}{2}\theta + mg\frac{L}{2} = 0$ X

$\frac{2}{3} L\ddot{\theta} - \frac{g}{2}\theta + \frac{g}{2} = 0$



$\Rightarrow \omega_n = \sqrt{\frac{+g/2}{2/3\, L}} = \sqrt{\frac{+g}{2}\times\frac{3}{2L}} = \sqrt{\frac{3g}{4L}}$ X

$\frac{\pi}{2} = \theta + \alpha + \frac{\pi}{4} - \theta$



$\theta = \frac{\pi}{4}$

$$\circlearrowleft + \sum M_\theta = 0 \Leftrightarrow mg\frac{L}{2}\sin\left(\frac{\pi}{4}+\theta\right) - mg\frac{L}{2}\sin\left(\frac{\pi}{4}-\theta\right) - I\ddot{\theta} = 0$$

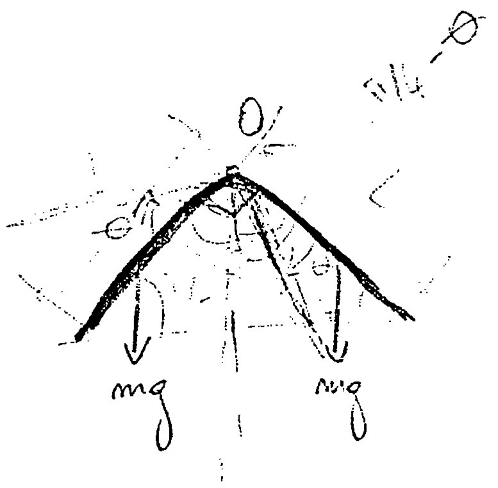
$$mg\frac{L}{2}\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\theta\right) - mg\frac{L}{2}\left(\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\theta\right) - I\ddot{\theta} = 0$$

$$I = 2\int r^2 dm = 2\int_0^L r^2 \rho A\, dr = 2\rho A \left.\frac{r^3}{3}\right|_0^L = \frac{2}{3}mL^2$$

$$\Leftrightarrow \frac{2}{3}mL^2\ddot{\theta} + mg\frac{L}{\sqrt{2}}\theta = 0$$

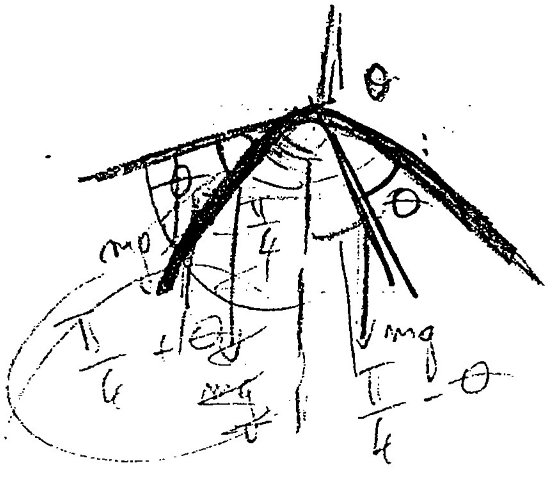
$$\frac{2}{3}L\ddot{\theta} + \frac{g}{\sqrt{2}}\theta = 0$$

$$\Rightarrow \omega_n = \sqrt{\frac{3g}{2\sqrt{2}L}}$$



$\frac{\pi}{2} = \theta + \frac{\pi}{4} - \theta + x$

$\Rightarrow x = \frac{\pi}{4}$



3) (20 pts) A counterrotating eccentric mass exciter operates on a structure having a mass of 181.4 kg. At a speed of 900 rpm, it was observed that the eccentric masses were at the top position and at that same instant, an upward deflection past the equilibrium position occurred with a magnitude of 21.6 mm. If the unbalance of each wheel of the exciter is 0.0921 kg.m, determine $\Rightarrow A_0$

a) the natural frequency of the structure

b) the damping constant of the system

2 spring in // $\Rightarrow K_{eq} = K_1 + K_2$

$\therefore K_{eq} = \frac{K}{2} + \frac{K}{2} = K$

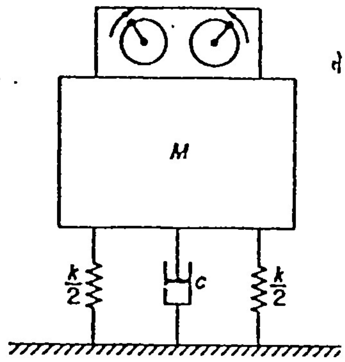
$\omega = \frac{900}{60} \times 2\pi = 94.247$ rad/sec.

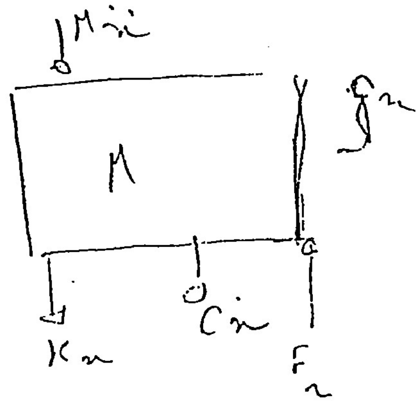
$= 0.0216$ m

$= 181.4$ Kg

$e = 0.0921$

2 wheels for the system





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$2 m_0 e \, \omega_n^2 \sin \omega_n t$

$= 2 \times 0.0921 \times (94.247)^2 \sin \omega_n t = 1636.15 \sin 94.247 t.$

$\therefore M\ddot{x} + Kx + C\dot{x} = F_n$

$181.4\, \ddot{x} + Kx + C\dot{x} = 1636.15 \sin 94.247 t.$

$\frac{\omega_n}{\omega_m} = \frac{94.247}{\sqrt{\frac{K}{181.4}}} = \sqrt{1-2\xi^2}$ (max. value $\Rightarrow$ r_p)

$\frac{(94.247)^2 \times 181.4}{K} = 1-2\xi^2 \quad \therefore K = \frac{1611284.957}{1-2\xi^2}$

$= \frac{2 m_0 e}{M} \times \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{2 \times 0.0921}{181.4} \times \frac{(1-2\xi^2)}{\sqrt{(1-1+2\xi^2)^2 + (2\xi\sqrt{1-2\xi^2})^2}}$

$0.0216 = 1.015 \times 10^{-3} \times \frac{(1-2\xi^2)}{}$ $\quad \therefore 21.27 \times (4\xi^2 - 4\xi^4)^{\frac{1}{2}}$

$452.48(4\xi^2 - 4\xi^4) = 1 + 4\xi^4 - 4\xi^2$

$1809.93\xi^2 - 1809.93\xi^4 = 1 + 4\xi^4 - 4\xi^2$

$1813.93\xi^2 - 1813.93\xi^4 - 1 = 0$

let $b = \xi^2$

$-1813.93b^2 + 1813.93b - 1 = 0$

$b = 5.51\times10^{-4}$; $b = 0.999$

$\Rightarrow \xi_1 = 0.02342$; $\xi_2 = 0.999\ldots$ since $\xi < \frac{1}{\sqrt{2}} = 0.707$

$K = \dfrac{1611284.957}{1 - 2\times(0.02341)^2} = 1613062.038$

$\omega_n = \sqrt{\dfrac{K}{M}} = \sqrt{\dfrac{1613062.038}{181.34}} = 94.314$ rad/sec

$\xi = \dfrac{C}{2\omega_n \times M} \Rightarrow C = \xi\, 2\omega_n \times M$

OK

$C = 0.02342 \times 2 \times 94.314 \times 181.34 = 102.81$

good