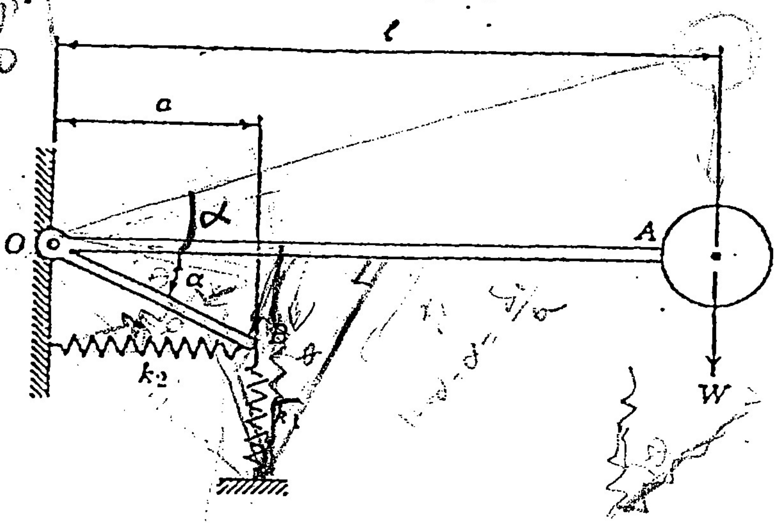
Name:

MEN 330
EXAM 1
FALL 2004

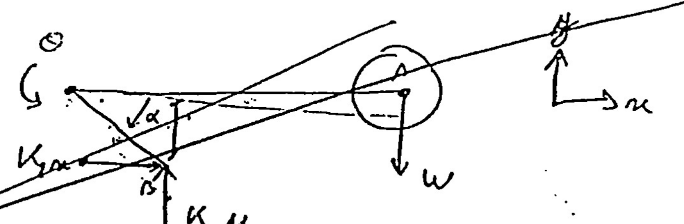
48

1) (20 pts) For recording vertical vibrations, the instrument shown in the figure is used, in which a rigid frame *AOB* carrying the weight *W* can rotate about an axis through *O* perpendicular to the plane of the figure.
Determine the angular frequency of small vertical vibrations of the weight, neglecting the masses of the frame and springs.



4

$\delta_2 = x_2$



$$\Sigma M_o = J_o\ddot{\theta}$$

$$W \times l - K_1 y a - K_2 x \times a \tan\alpha = J\ddot{\theta}$$

$$W \times l - K_1 y a - K_2 y\theta \times a \tan\alpha = J\ddot{\theta}$$

$x = y \sin\theta$ for small vibration

$x = y\theta$

$$\Sigma M_o = 0 \rightarrow I\ddot{\theta} + k_1 x_1 (a) + k_2 x_2 (a) = 0$$

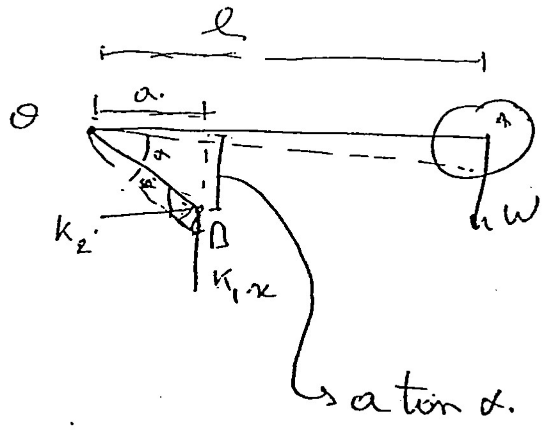
$x_1 \rightarrow \theta a$, $x_2 \rightarrow \theta a$

$\alpha = \tan\alpha = \sin\alpha$

$$\rightarrow I\ddot{\theta} + k_1 \theta a^2 + k_2 \theta a^2 = 0$$

$$\rightarrow \frac{W}{g} l^2 \ddot{\theta} + k_1 a^2 \theta + k_2 a^2 \theta = 0$$

$$m l^2 \ddot{\theta} + \theta (k_1 a^2 + k_2 a^2) = 0$$



$\Sigma F_x = m\ddot{x}$

$+W - K_1 x = m\ddot{x}$

$m\ddot{x} + K_1 x + W = 0$

$\cancel{\ddot{x} + \frac{K_1}{m}x + g = 0}$

$\cancel{\omega^2 = \frac{K_1}{m}} \to \omega = \sqrt{\frac{K_1}{m}}$

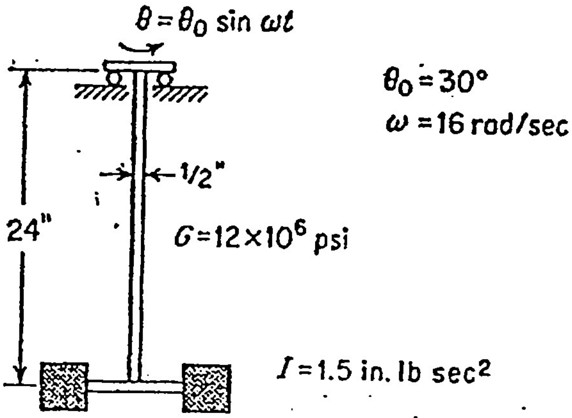
$W = \sqrt{\frac{K_1}{m}}$

~~$\Sigma M_O = 0$ at equilibrium.~~

$\cancel{+W \times l - K_1 x a - K_2 x a \tan\alpha = 0}$

$\cancel{W = \frac{(K_1 + K_2 \tan\alpha) a}{l}}$

2) (15 pts) A mechanical agitator is designed so that the power end rotates in a predetermined simple harmonic motion $\theta = \theta_o \sin \omega t$. The machine operates with negligible damping. Find the maximum shear stress in the shaft and the maximum angular displacement amplitude of the paddle for steady-state motion.



5

SHM; $\theta = \theta_o \sin\omega t \Rightarrow \theta = 30 \sin 16t$.

No damping.

Find τ_{max} ??

max angular displacement amplitude ??

Stiffness of the bar: $K = \frac{G \times I}{L} = \frac{12 \times 10^6 \times 1.5}{24} = 750 \times 10^3$ N/m

$= 750$ KN/m

$J\ddot{\theta} + K\theta = \theta_o \sin\omega t$

angular displacement amplitude: $\theta = \frac{\theta_o}{\omega_n^2 - \omega^2} = \frac{30}{\left(\frac{K}{J}\right)^2 - 16^2}$

$\tau = \frac{T.r}{J}$ max shear stress correspond to max T.

3) (15 pts) A typical unbalanced machine has a machine mass of 120 kg, a mount stiffness of 800 kN/m, and a damping constant of 500 kg/s. The amplitude of the out-of-balance force is measured to be 374 N at a running speed of 3000 rev/min. Determine the amplitude of motion due to the out-of-balance.

$m = 120\ Kg$

$K = 800\ KN/m$

$C = 500\ Kg/s$

$F_0 = 374\ N$

15

$\omega_r = 3000\ rev/min = 3000\,rev \times \frac{2\pi \cdot rd}{1\,rev} \times \frac{1\,min}{60\,s}$

$= 100\pi\ rd/s$

$m\ddot{x} + c\dot{x} + Kx = m_0 e\, \omega_r^2 \sin \omega_r t$. is the eq of motion of the system.

$\Rightarrow \omega_n^2 = \frac{K}{m}$

$\Rightarrow \omega_n = \sqrt{\frac{800 \times 10^3}{120}} = 81.7\ rd/s$ ✓

the amplitude of the out-of-balance force is:

$m_0 e \omega_r^2 = 374\ N$

$m_0 e = \frac{374}{\omega_r^2} = \frac{374}{(100\pi)^2} = 3.8 \times 10^{-3}$ ✓

The solution of the eq of the motion of unbalanced is given by:

$x_p(t) = X \sin(\omega_r t - \phi)$.

where, $X = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$