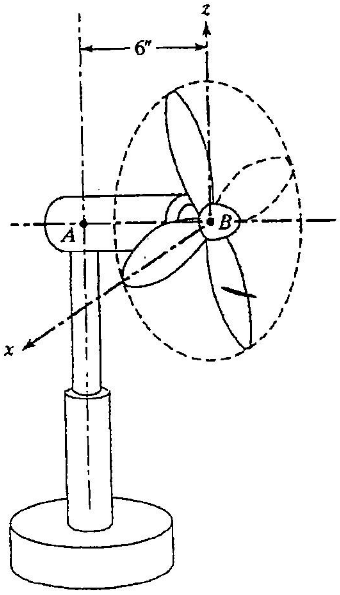
Name:

MEN 330
EXAM 1
Summer 2006

64

1) (15 pts) One of the blades of an electric fan is removed (as shown by the dotted lines in the figure). The steel shaft AB, on which the blades are mounted, is equivalent to a uniform shaft of diameter $1in$ and length $6in$. Each blade can be modeled as a uniform slender rod of weight $2lb$ and length $12in$. Determine the natural frequency of vibration of the remaining three blades about the y-axis.



10

$$W_n = \sqrt{\frac{K_{eq}}{I}}$$

$$\boxed{K_{eq} = \frac{GJ}{L}}$$

$$\boxed{G = 11 \times 10^6 \, psi}$$

$$\boxed{L = 6 \, in}$$

$$J = \frac{\pi}{2}\left(\frac{1}{2}\right)^4 = \frac{\pi}{32}$$

$$\boxed{J = 0.098 \, in^4}$$

$$K = \frac{11\times10^6 \times 0.098}{6} = 18\times10^4 psi$$

$$\boxed{K = 18\times10^4 \ \cancel{psi}\ lb/in}$$ ok

I of the propeller $= \cancel{3}\, mL^2 = 2\times(12)^2 = 288\, psi$

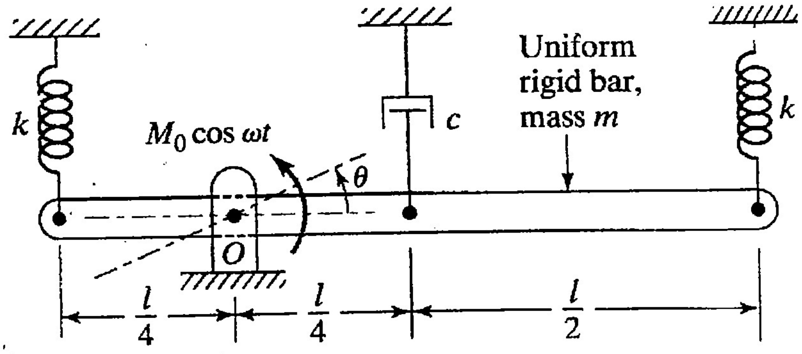
$$\boxed{I = 288\ \cancel{psi}\ lb.in^2} \times 3 \Rightarrow \boxed{I = 864\ lb.in^2}$$

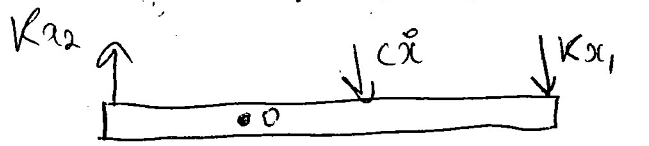
$$W_n = \sqrt{\frac{18\times10^4}{\cancel{288}\ 864}} = \cancel{\sqrt{625}} = \cancel{25} = 14.43\ rd/s$$

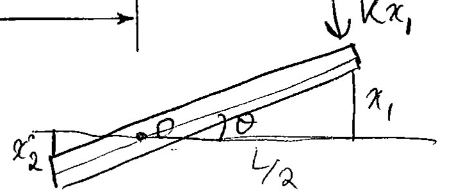
$$\cancel{\boxed{W_n = 25\ rd/s}}$$

$$\boxed{W_n = 14.43\ rd/s}$$

2) (15 pts) Derive the equation of motion and find the steady-state response of the system shown for rotational motion about point O. The following are given: $k = 5000\,N/m$, $l = 1m$, $c = 1000\,N.s/m$, $m = 10\,kg$, $M_0 = 100\,N.m$, $\omega = 1000\,rpm$.







ΣM_o $\curvearrowleft +$: $-I_o\ddot{\theta} - Kx_2\left(\frac{L}{4}\right) - Kx_1\left(\frac{3L}{4}\right) - c\dot{x}\left(\frac{L}{4}\right) + M_o\cos\omega t = 0$

$I_o = \frac{1}{12}mL^2 + m\left(\frac{L}{4}\right)^2 = \frac{1}{12}mL^2 + \frac{1}{16}mL^2$ 12

$\Rightarrow \boxed{I_o = \frac{7}{48}mL^2}$ ✓ $I_o = \frac{7}{48}\times 10\times 1^2 = 1.46\ Kg\cdot m$

$\boxed{I_o = 1.46\ Kg\cdot m}$

$\boxed{\omega_n = \frac{1000\times 2\pi}{60} = 104.7\ rad/s}$ $x_1 = \frac{L}{2}\sin\theta = \frac{1}{2}\theta$ $x_2 = \frac{L}{4}\sin\theta = \frac{1}{4}\theta$

$\boxed{c = 1000\ N\cdot s/m}$ $x = \frac{1}{2}\theta \Rightarrow \dot{x} = \frac{1}{2}\dot{\theta}$

$\Rightarrow 1.46\ddot{\theta} + 5000\times\frac{1}{16}\theta + 5000\times\frac{3}{8}\theta + 1000\times\frac{1}{8}\dot{\theta} = 100\cos 104.7t$

$\boxed{1.46\ddot{\theta} + 125\dot{\theta} + 2187.5\theta = 100\cos 104.7t}$

$\theta(t) = \theta_o\cos(\omega_m t + \phi)$ $\omega_m = \sqrt{\frac{2187.5}{1.46}} = 38.7\ rad/s$ X

$$\theta_0 = \frac{m_0}{\left[(w_m^2 - w_{dr}^2)^2 + (2\,\delta\, w_m w_{dr})^2\right]^{\frac{1}{2}}}$$

$$m_0 = \frac{M_0}{I_0} = \frac{100}{1.46} = 68.5\ N.m$$

$$\boxed{m_0 = 68.5\ N.m}$$

$$\boxed{w_m = 38.7\ rd/s} \qquad \boxed{w_{dr} = 104.7\ rd/s}$$

$$\delta = \frac{c}{2 I_0 w_m} = \frac{1000}{2 \times 1.46 \times 38.7} = 8.85$$

$$\boxed{\delta = 8.85}$$

replace:

$$\cancel{\frac{68.5}{[(\ldots}} \Rightarrow \boxed{\theta_0 = 9.47 \times 10^{-4}\ rd}$$

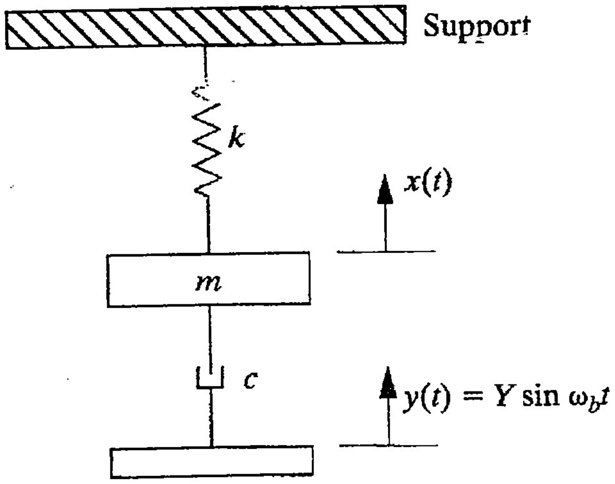
$$\cancel{\theta_{(t)} = 9.47 \times 10^{-4} \cos}$$

$$\phi = \tan^{-1}\left(\frac{2\,\delta\, w_m w_{dr}}{w_m^2 - w_{dr}^2}\right)$$

$$\boxed{\phi = -82.48\ rd}$$

$$\therefore \boxed{\theta_{(t)} = 9.47 \times 10^{-4} \cos(38.7t - 82.48)}$$

3) (20 pts) For the system shown below, derive an expression for the force transmitted to the support in steady-state, in terms of m, Y, ζ, ω_n, and ω_b.



$$\frac{F_T}{KY} = r^2 \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2} \right]^{1/2}$$ 10

$$r = \frac{\omega_b}{\omega_n} \qquad \omega_n = \sqrt{\frac{K}{m}} \Rightarrow \omega_n^2 = \frac{K}{m}$$

$$\boxed{K = \omega_n^2 m}$$

$$\therefore F_T = \omega_n^2 \cdot m \cdot Y \cdot \frac{\omega_b}{\cancel{\omega_n}} \left[\frac{1 + 4\zeta^2 \frac{\omega_b^2}{\omega_n^2}}{\left(1 - \frac{\omega_b^2}{\omega_n^2}\right)^2 + 4\zeta^2 \frac{\omega_b^2}{\omega_n^2}} \right]^{1/2}$$

$$\boxed{F_T = \omega_n \omega_b m Y \left[\frac{1 + 4\zeta^2 \frac{\omega_b^2}{\omega_n^2}}{\left(1 - \frac{\omega_b^2}{\omega_n^2}\right)^2 + 4\zeta^2 \frac{\omega_b^2}{\omega_n^2}} \right]^{1/2}}$$