Name:

MEN 330
EXAM 1
SPRING 2005

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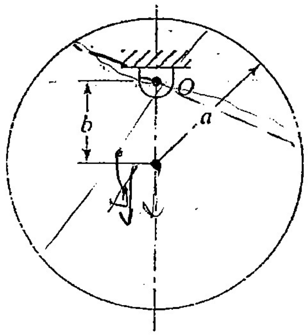
1) (20 pts) Find the natural frequency of oscillation of the system made up of a uniform circular disc pivoted at point O.



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$$\Sigma M_o = I_{eq}\ddot{\theta} \Rightarrow I_{eq}\ddot{\theta} = -k_t\theta \Rightarrow I_{eq}\ddot{\theta} + k_t\theta = 0$$ X

No spring here

$$\cancel{I_{eq} = \frac{1}{2} m a^2 + m (b)^2}$$

$$\Rightarrow \omega_n = \sqrt{\frac{k_t}{I_{eq}}}$$

$$I_{eq} = \frac{WD^2}{8g} + \frac{W}{g} b^2 = \frac{WD^2 + 8Wb^2}{8g}$$

$$\Rightarrow \omega_n = \sqrt{\frac{k_t}{\frac{W(D^2+8b^2)}{8g}}}$$

$$\Rightarrow \omega_n = \sqrt{\frac{8g\,k_t}{W(D^2+8b^2)}} = \sqrt{\frac{8g\,k_t}{W(a^2+8b^2)}}$$

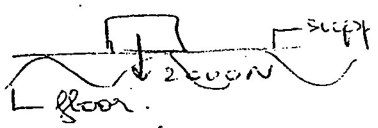
$$k_t = \frac{GJ}{L} = \frac{\pi G d^4}{32L}$$

$$\Rightarrow \omega_n = \sqrt{\frac{8g\,\frac{\pi G d^4}{32L}}{W(a^2+8b^2)}}$$ X

$$\omega_n = \sqrt{\frac{g\pi G d^4}{4L(W(a^2+8b^2)}}$$, d = diameter of shaft X

2) (15 pts) A machine weighing 2000 N rests on a support. The support deflects about 5 cm as a result of the machine. The floor under the support is somewhat flexible and moves, because of the motion of a nearby machine, harmonically at resonance with an amplitude of 0.2 cm. Assume a damping ratio of $\varsigma = 0.01$, and calculate the amplitude of the transmitted force $\Rightarrow F_T$ and the amplitude of the transmitted displacement $\Rightarrow Z$.

$$* \; F_T = KYr^2\left[\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}\right]^{1/2} \quad (1)$$



floor moves at resonance $\Rightarrow r = 1$.

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$\zeta = 0.01$

$Y = 0.2$ cm

* The support deflect about 5cm as a result of th machine resting on the support so, $k = \frac{2000N}{5\times10^{-2}\,m}$

$\Rightarrow k = 40\,000 \frac{N}{m}$

$$(1): \; F_T = 40000\frac{N}{m}\left(0.2\times10^{-2}\,m\right)(1^2)\left[\frac{1+(2\times0.01)^2}{0+(2\times0.01)^2}\right]^{1/2}$$

$$\Rightarrow F_T = 80N\left[\frac{1.0004}{4\times10^{-4}}\right]^{1/2} \Rightarrow \boxed{F_T = 4000.7999\,N}$$

F_T is the amplitude of the transmitted force.

Z is the amplitude of the transmitted displacement with,

$$Z = Y\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$\Rightarrow Z = 0.2\times10^{-2}(m)\frac{1}{\sqrt{0+(2(0.01))^2}}$$

$$\Rightarrow Z = \frac{0.2\times10^{-2}(m)}{2\times0.01}$$

$$\Rightarrow \boxed{Z = 0.1\,m.}$$

a) (15 pts) An electric motor weighing 750 *lb* and running at 1800 *rpm* is supported on $\boxed{\text{four}}$ steel helical springs, each of which has eight active coils with a wire diameter of 0.25 *in* and a coil diameter of 3 *in*. The rotor has a weight of 100 *lb* with its center of mass located at a distance of 0.01 *in* from the axis of rotation. Find the amplitude of the $\boxed{\text{force}}$ transmitted to the base of the motor.

$n = 8$. e

$G = 11 \times 10^6$ psi.

* $\omega_{motor} = 1800$ rpm $= \dfrac{1800 \times 2\pi}{60} = 188.4955$ rad/s.

* $m_o e = \dfrac{100\,lb}{32.2 \times 12\, in/s^2} \times 0.01$ in $= 2.588 \times 10^{-3}$ $lb_m \cdot in$.

* $F_o = m_o e \omega^2 = 91.953$

* $K_{helical\ spring} = \dfrac{G d^4}{8 n D^3} = \dfrac{11 \times 10^6 (lb/in^2)\,(0.25\,in)^4}{8(8)(3\,in)^3} = k$

As Springs are in parallel so, $k_{eq.} = 4k = 99.465 \dfrac{lb}{in}$

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$$X = \frac{m_o e r^2}{M\left[(1-r^2)^2 + (2\zeta r)^2\right]^{1/2}}$$

$\zeta = 0$; $r = \dfrac{\omega}{\omega_n} = \dfrac{188.4955}{\sqrt{\dfrac{K_{eq}}{M_{motor}}}} = \dfrac{188.4955}{\sqrt{\dfrac{99.465}{\frac{750}{32.2 \times 12}}}} \simeq 26.33$

$$\Rightarrow X = \frac{1.794452492}{\frac{750}{32.2 \times 12}\left(1 - 26.332^2\right)}$$

$$X \simeq -1.3353 \times 10^{-3} \text{ in.}$$

$$\Rightarrow F_T = k_{eq} X r^2 \left[\frac{1}{(1-r^2)^2}\right]^{1/2}$$

$$F_T = 99.465\left(-1.3353 \times 10^{-3}\right) 26.332^2 \left[\frac{1}{1 - 26.332^2}\right]$$

$$\Rightarrow F_T = 0.133007 \text{ lb}$$

F_T is the amplitude of the force transmitted to the base of the motor.