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MEN 330
EXAM 1
FALL 2006

76

1) (15 pts) The hydrometer shown is used to measure the specific gravity of liquids. The mass of the float is $0.0372\,kg$ and the diameter of the cylindrical section protruding above the surface is $0.0064\,m$. Determine the period of oscillation of the float in a liquid of specific gravity 1.20. Note that the specific weight of a liquid (in units of $\frac{force}{volume}$) is equal to the specific gravity of the liquid multiplied by $9806\frac{N}{m^3}$.

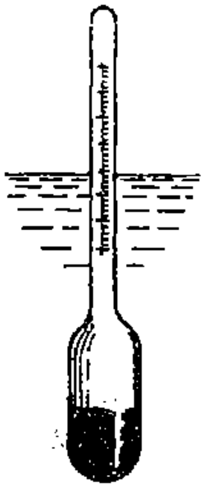
$m_f = 0,0372\,Kg.$

$d = 0,0064\,m.$

$sg = 1,2.$

specific weight γ

$\gamma = sg \times 9806.$



15

mass of the hydrometer is $m_f = 0,0372\,Kg$.

Restoring force ~~of~~ in the liquid : $F = \rho g \times V_{displaced}$

units of $\rho g = \frac{kg}{m^3} \times \frac{m}{s^2} = \frac{kg}{m^2 s^2} = \frac{kg\,m}{s^2} \times \frac{1}{m^3} = \frac{N}{m^3} \Rightarrow \rho g = \gamma.$

$F = \gamma \times V_{displaced} = 1,2 \times 9806 \times A \times x$

$= 1,2 \times 9806 \times \pi r^2 \times x$

$= 1,2 \times 9806 \times \pi (0,0032)^2 \times x$

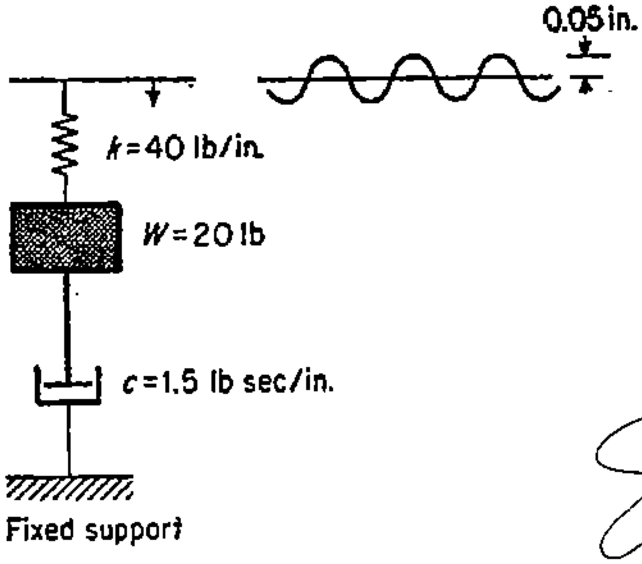
$= 0,37855\,x$

$\Rightarrow 0,0372\,\ddot{x} + 0,37855\,x = 0$

$w_n = \sqrt{\frac{0,37855}{0,0372}} = 3,19\,rad/s.$

$T = \frac{2\pi}{w} = 1,97\,s$

2) (20 pts) The upper support of the system shown moves harmonically with an amplitude of $0.05 in$ and with a frequency equal to the undamped natural frequency of the system. Calculate the amplitudes of the spring and damping forces.



8

K = 40 lb/in   W = 20lb   c = 1,5lb-sec/in.   y(t) = Y sin wbt

wb = wn   y(t) = 0,05 sin wbt.

=> r = 1

$K(x-y)$

m

$m\ddot{x}$

$c(\dot{x}-\dot{y})$

$c\dot{x}$ only, no $\dot{y}$ here, so does not [illegible] fixed

equation of motion.

$K(x-y)+c(\dot{x}-\dot{y}) - m\ddot{x} = Y \sin wbt.$

$wb = wn = \sqrt{\frac{K}{m}} = \sqrt{\frac{40}{20/386,4}} = 27,8$ rad/s. $\zeta = \frac{c}{2mwn} = \frac{1,5}{2\times\frac{20}{386,4}\times 27,8} = 0,52.$

$$\frac{X}{Y} = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}} = \sqrt{\frac{1+(2\times 0,52)^2}{(2\times 0,52)^2}} = 1,3873$$

$X = 0,05 \times 1,3873$

$X = 0,07$ in.

$y(t) = 0,05 \sin 27,8t$ $y'(t) = wY \cos wbt$ $y''(t) = -w^2 Y \sin wbt$

~~$F_s = K x(t)$~~

Spring force.

$F_{sp} = K(x - y)$

~~$F_s = K(0{,}07 \sin 27{,}8t - 0{,}08)$~~

$x_{p1}(t) = C Y \omega_b \cos \omega_b t$

$x_{p2}(t) = K Y \sin \omega_b t$

$$x_p(t) = \omega_n Y \sqrt{\left[\frac{\omega_n^2 + (2\zeta \omega_e)^2}{(\omega_n^2 - \omega_b^2) + (2\zeta \omega_n \omega_b)^2}\right]} \cos(\omega_e t - \phi_1 - \phi_2)$$

Damping force:

$F_d = c(\dot{x} - \dot{y})$

$F_{s} = K(0{,}02) = 0{,}8$.

$F_d = 1{,}5(\dot{x} - \dot{y})$

3) (15 pts) A steel beam ($E = 30000000\,psi$) is simply-supported, has a total length $L = 12\,ft$, and a moment of inertia $I = 57.6\,in^4$. It carries at its mid-span an electric motor of weight $W = 1000\,lb$ that runs at $1800\,rpm$. Due to the unbalance, the rotor sets up a rotating centrifugal force of magnitude $P = 500\,lb$. Neglecting the weight of the beam, calculate the amplitude of vibration in steady-state. It is known that when a unit static load is applied at mid-span, the corresponding static deflection of the beam at mid-span is $\frac{L^3}{48EI}$.

$E = 30\,000\,000\,psi \quad L = 12\,ft = 144\,in \quad I = 57{,}6\,in^4 \quad W = 1000\,lb$

$P = 500\,lb$

$$K = \frac{48EI}{l^3} = \frac{48 \times 30000000 \times 57{,}6}{144^3} = 27777{,}77.$$

$$w_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{27777{,}77}{1000/386{,}4}} = 103{,}6\,rad/s.$$

$$w_r = 1800\frac{rev}{min} \times \frac{2\pi\,rad}{rev} \times \frac{1\,min}{60\,s} = 188{,}5\,rad/s.$$

$$r = \frac{w_r}{w_n} = \frac{188{,}5}{103{,}6} = 1{,}82.$$

15

$$X = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

No damping $\zeta = 0 \Rightarrow X = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2}} = \frac{m_0 e}{m} \frac{r^2}{1-r^2}$

$P = m_0 e\, w^2 \Rightarrow 500 = m_0 e\, w_r^2$

$$m_0 e = \frac{500}{188{,}5^2} = 0{,}01407$$

$m_{motor} = \frac{1000}{386{,}4} = 2{,}588\,lb/in$

$$\Rightarrow X = \frac{0{,}01407}{2{,}588} \times \frac{1{,}82^2}{(1-(1{,}82)^2)} = 0{,}00778\,in.$$