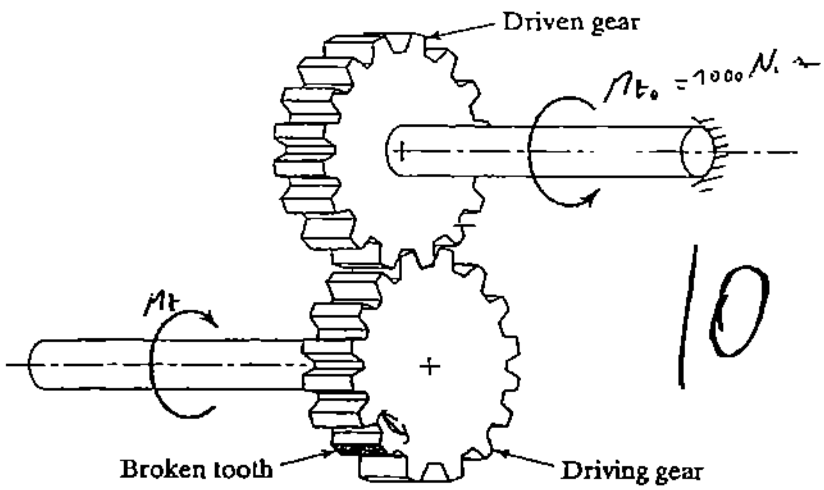
29, 37, 1, 8, 3

Name: Elias Najem

88

MEN 330
EXAM 2
FALL 2004

1) (15 pts) Suppose that during the engagement of two gears as shown under steady state conditions, one of the 16 teeth on the driving gear is missing. Let the speed of the driven gear be 1000 *rpm*. If the amplitude of the torque delivered to the driven gear is $M_{t0} = 1000\,N.m$, plot the torque loading M_t as a function of time for the first two cycles and suggest a method to determine the response of the driven gear under that loading **but <u>do not</u> attempt to solve the problem.**



10

$T = \frac{2\pi}{\omega_r} = \frac{2\pi}{100}$

$= 6.28$

Fourier series

$\frac{T}{16} = 3,92 \times 10^{-4}$

$\left(T - \frac{T}{16}\right) = X = (6{,}2$

15 teeth

16 teeth $\rightarrow 2\pi$ rad

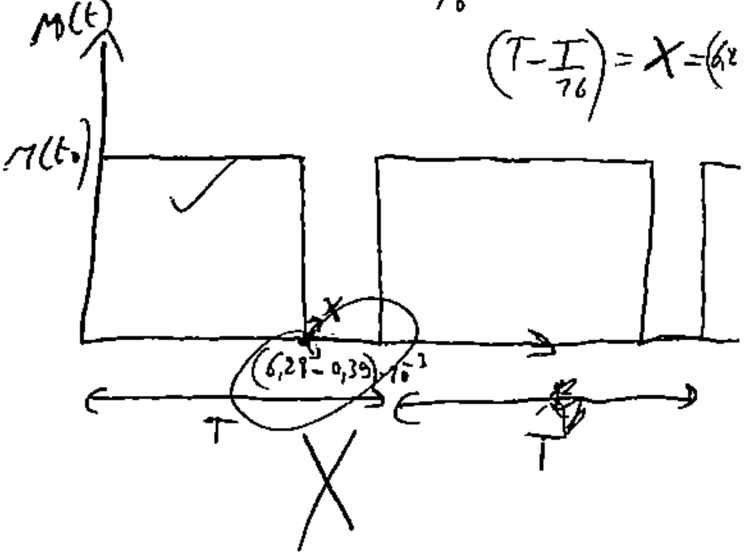
15 teeth $\rightarrow 5.89$ rad

1 tooth $\rightarrow 0.3927$

104.7 rad $\rightarrow$ 1 s

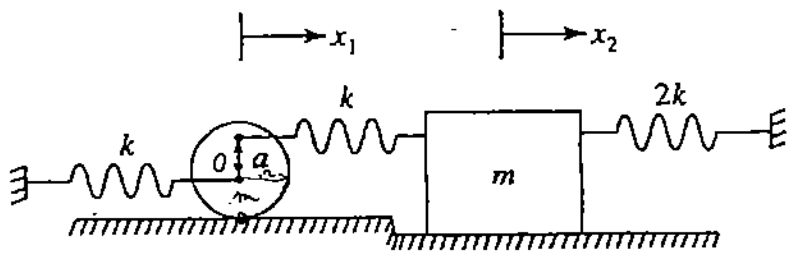
5.89 rad $\rightarrow$ 0.05625 s

0.3927 $\rightarrow$ 0.00375 s



correction:

2) (20 pts) The cylinder shown has a mass m, a radius r, is attached to the left spring at its center, and to the right spring at a point located a distance a above its center. Given $m = 2\,kg, r = 0.1\,m, a = 0.075\,m, k = 4000\,N/m$, determine the natural frequencies of the system for small oscillations. The cylinder rolls without slipping and the mass slides on a frictionless support.



$x_1 = r\theta_1$

$x' = (r+a)\theta$

$x' = x_1 + a\theta_1$

$x' = x_1 + \frac{a}{r}x_1$

$\rightarrow k(x_2 - x')$

14

~~$\theta = \frac{x_1}{a}$~~ $\Rightarrow$ ~~$x_1 = a\theta$~~. $k(x_2-(r+a)\theta)$.

$\circlearrowleft^+ \sum M_0 = J_0\ddot{\theta} \Rightarrow k[x_2-(r+a)\theta](r+a) - kx_1 r - I_0\ddot{\theta} = 0$

$-k\cancel{(x_1-x_2)a = \frac{1}{2}mr^2\ddot{\theta} \Rightarrow -\frac{1}{2}mr^2\ddot{\theta} + k(x_1-x_2)a = 0}$

$I_0 = \frac{3}{2}mr^2$, $\boxed{x_1 = r\theta}$ $\cancel{-\frac{1}{2}mr^2\ddot{\theta} - ka^2\theta + kx_2a = 0}$

$-k[x_2-(r+a)\theta] - 2kx_2 - m\ddot{x}_2 = 0.$

$\sum F_x = m\ddot{x} \Rightarrow$ $\cancel{-m\ddot{x} - 2kx_1 + kx_1 - kx_2 = 0}$

$\cancel{-m\ddot{x} - 3kx_1 + ka\theta = 0}$

$$\begin{pmatrix} \frac{3}{2}m & 0 \\ 0 & m \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} k\left(2+\frac{2a}{r}+\frac{a^2}{r^2}\right) & -k\left(1+\frac{a}{r}\right) \\ -k\left(1+\frac{a}{r}\right) & 3k \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\det(-M\omega^2 + K) = 0.$$

$$\det\begin{pmatrix} m\omega^2 - 3K & Ka \\ Ka. & \frac{1}{2} m r^2 \omega^2 - K a^2 \end{pmatrix} = 0$$

$$(m\omega^2 - 3K)\left(\frac{1}{2} m r^2 \omega^2 - K a^2\right) - \left(K^2 a^2\right) = 0,$$

$$\frac{1}{2} m^2 r^2 \omega^4 - m\omega^2 K a^2 - \frac{3K}{2} m r^2 \omega^2 + 3K^2 a^2 - K^2 a^2 = 0.$$

$$\frac{1}{2} m^2 r^2 \omega^4 - \left(m K a^2 + \frac{3K}{2} m r^2\right)\omega^2 + 2K^2 a^2 = 0.$$

$$\frac{1}{2} m^2 r^2 X^2 - \left(m K a^2 + \frac{3K}{2} m r^2\right) X + 2k^2 a^2 = 0.$$

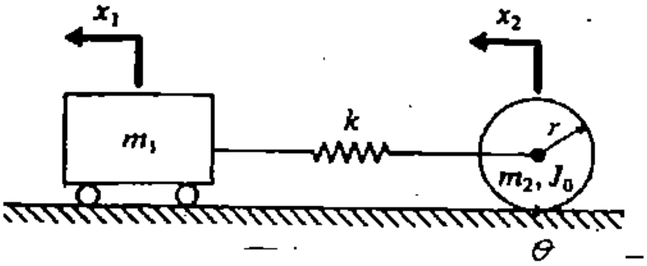
$$\cancel{\tfrac{1}{2}}\, 4 \cdot 0{,}02\, X^2 - \left(\cancel{2 \times 1{,}0 \times 0 \times 5}^{\,45} + 720\right) X + 180000 = 0$$

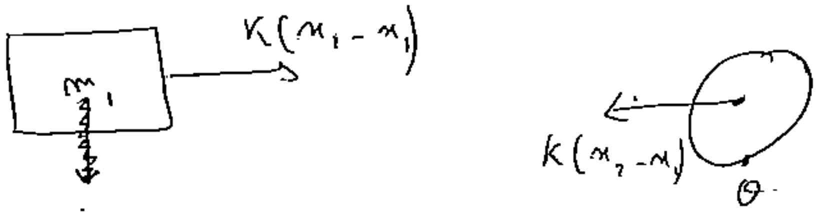
$$X_1 = 6956{,}18 = \omega_1^2 \rightarrow \omega_1 = 83{,}40 \text{ rad/s}$$

$$X_2 = 1293{,}8 = \omega_2^2 \rightarrow \omega_2 = 35{,}96 \text{ rad/s}$$

Do not have Previous exam

3) (15 pts) Determine the natural frequencies of the system for small oscillations under no slip conditions.





$\Sigma F_{x_1} = m_1 \ddot{x}_1$ $\qquad$ $\Sigma F_{x_2} = m_2 \ddot{x}_2$

$K(x_2 - x_1) = m_1 \ddot{x}_1$ $\qquad$ $K(x_2 - x_1) = m_2 \ddot{x}_2$

$m_1 \ddot{x}_1 - K x_2 + K x_1 = 0$ $\qquad$ $m_2 \ddot{x}_2 - K x_2 + K x_1 = 0$

$$M \ddot{x} + K x = 0$$

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} K & -K \\ -K & K \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = ($$

$$\det(-M\omega^2 + K) = 0$$

$$-M\omega^2 + K = \begin{pmatrix} -m_1\omega^2 & 0 \\ 0 & -m_2\omega^2 \end{pmatrix} + \begin{pmatrix} K & k \\ -K & k \end{pmatrix}$$

$$= \begin{pmatrix} -m_1\omega^2 + K & K \\ -K & -m_2\omega^2 + K \end{pmatrix}$$

$\cancel{-m_1\omega^2 K + K^2}$

$$(-m_1\omega^2 + K)(-m_2\omega^2 + K) + K^2 = 0$$

$$m_1 m_2 \omega^4 - m_1\omega^2 K - m_2\omega^2 K + K^2 + K^2 = 0$$

$$m_1 m_2 \omega^4 + (-m_1 - m_2) K \omega^2 + 2K^2 = 0$$

Let $W^2 = \omega^4$

$$m_1 m_2 W^2 + (-m_1 - m_2) K W + 2K^2 = 0$$

$$W =$$