Math 201 — Fall 2009–10
Calculus and Analytic Geometry III, sections 1–8, 24–26
Final Exam, February 1 — Duration: 2.5 hours

GRADES:

1 (15 pts)	2 (12 pts)	3 (10 pts)	4 (12 pts)	5 (14 pts)	6 (12 pts)	7 (15 pts)
8 (15 pts)	9 (25 pts)	10 (28 pts)	11 (14 pts)	12 (18 pts)	13 (10 pts)	Total/200

YOUR NAME:

YOUR AUB ID#:

PLEASE CIRCLE YOUR SECTION:

Section 1 Lecture MWF 3 Professor Makdisi Recitation F 11 Ms. Nassif	Section 2 Lecture MWF 3 Professor Makdisi Recitation F 2 Ms. Nassif	Section 3 Lecture MWF 3 Professor Makdisi Recitation F 4 Ms. Nassif	Section 4 Lecture MWF 3 Professor Makdisi Recitation F 9 Ms. Nassif
Section 5 Lecture MWF 10 Professor Raji Recitation T 11 Professor Raji	Section 6 Lecture MWF 10 Professor Raji Recitation T 3:30 Ms. Itani	Section 7 Lecture MWF 10 Professor Raji Recitation T 8 Ms. Itani	Section 8 Lecture MWF 10 Professor Raji Recitation T 2 Ms. Itani
Section 24 Lecture MWF 2 Professor Tlas Recitation F 11 Dr. Yamani	Section 25 Lecture MWF 2 Professor Tlas Recitation F 12 Dr. Yamani	Section 26 Lecture MWF 2 Professor Tlas Recitation F 3 Professor Tlas	

INSTRUCTIONS:

1. **Write your NAME and AUB ID number, and circle your SECTION above.**
2. **Solve the problems inside the booklet. Explain your steps precisely and clearly to ensure full credit. Partial solutions will receive partial credit.**
3. **You may use the back of each page for scratchwork OR for solutions. There are five blank sheets at the end, for extra scratchwork or solutions. If you need to continue a solution on another page, INDICATE CLEARLY WHERE THE GRADER SHOULD CONTINUE READING.**
4. **Closed book and notes. NO CALCULATORS ALLOWED. Turn OFF and put away any cell phones.**

GOOD LUCK!

An overview of the exam problems. Take a minute to look at all the questions, THEN solve each problem on its corresponding page INSIDE the booklet.

1. (5 pts each part, 15 pts total) Determine whether each of the following series converges or diverges:

$$\text{(a)} \sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{\sqrt{n^5+1}} \qquad \text{(b)} \sum_{n=1}^{\infty} \tan^{-1} n \qquad \text{(c)} \sum_{n=1}^{\infty} (n+1)(0.5)^n$$

2. (12 pts) Find the interval of convergence of the following series. Remember to test the endpoints.

$$\sum_{n=1}^{\infty} \frac{(x-5)^n}{\sqrt{n}\,3^n}$$

3. (5 pts each part, 10 pts total)

a) Find the first four nonzero terms of the Taylor series for e^x and $\cos 3x$ centered at $a = 0$.

b) Compute the limit: $\displaystyle \lim_{x\to 0} \frac{e^x - 1 - x}{1 - \cos 3x}$.

4. (6 pts each part, 12 pts total)

a) Find the second-order Taylor polynomial of the function $f(x) = \sqrt{x}$, **centered at** $x = 100$.

b) Using your answer P_2 above, give an approximate value for $\sqrt{99}$ and estimate the error.

5. (7 pts each part, 14 pts total) We are given a function $f(x, y)$ satisfying:

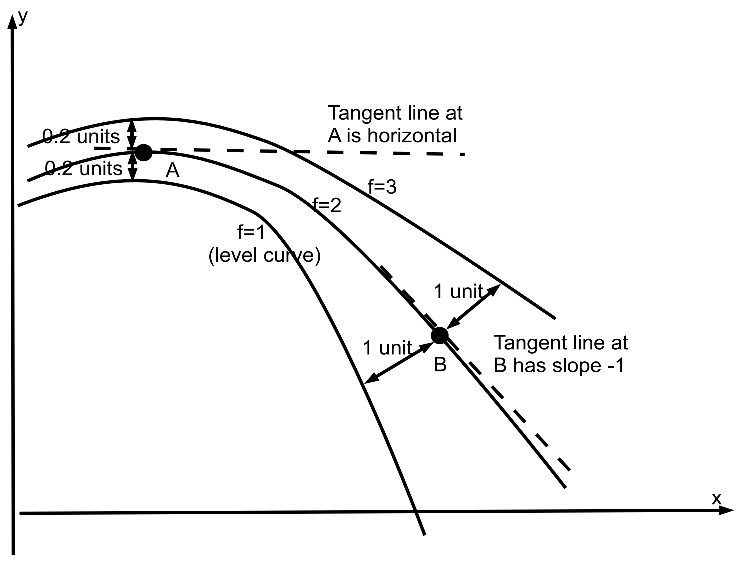
$$\vec{\nabla} f \Big|_{(1,2,3)} = (4, -5, 6), \qquad \vec{\nabla} f \Big|_{(2,4,6)} = (8, -9, 10), \qquad \vec{\nabla} f \Big|_{(2,0,1)} = (2, 0, 2).$$

a) Which of the following differences do you expect to be largest, and why?

$$f(1.1, 2.1, 3.1) - f(1, 2, 3), \qquad f(2.1, 4.1, 6.1) - f(2, 4, 6), \qquad f(2.1, 0.1, 1.1) - f(2, 0, 1).$$

b) Let $g(r, s) = f(r - s, r + s, 2rs)$. Find $\displaystyle \frac{\partial g}{\partial s}\Big|_{(r,s)=(3,1)}$.

6. (4 pts each part, 12 pts total) Some level curves of a function $f(x, y)$ are drawn below. **The rest of problem 6 is on page iii**.



6, continued. a) Give two **unit** vectors $\vec{u}_A, \vec{u}_B$ such that $\vec{u}_A$ points in the same direction as $\vec{\nabla} f\Big|_A$, and similarly $\vec{u}_B$ points in the same direction as $\vec{\nabla} f\Big|_B$.

b) Based on the figure, what is a reasonable approximation to the directional derivatives

$$D_{\vec{u}_A} f\Big|_A, \qquad D_{\vec{u}_B} f\Big|_B?$$

c) Deduce a reasonable approximation to each of the vectors $\vec{\nabla} f\Big|_A$ and $\vec{\nabla} f\Big|_B$.

7. (15 pts) Find and classify the critical points of the function $f(x,y) = \dfrac{x^4}{12} - 3xy + \dfrac{3y^2}{2}$.

8. (15 pts total) Let $f(x,y,z) = 8x + y + z$, and consider the surface S given by $xyz = 1$ in the first octant only:

$$S = \{(x,y,z) \mid xyz = 1,\ x,y,z > 0\}.$$

a) (12 pts) Use Lagrange multipliers to find the minimum value of f on the surface S.

b) (3 pts) Show that f does not have a maximum value on S. (Why are the values of f not bounded above on S?)

9. (5 pts each part, 25 pts total) Consider the integral

$$I = \int_{x=0}^{1} \int_{y=0}^{x} (x+y)\, dydx.$$

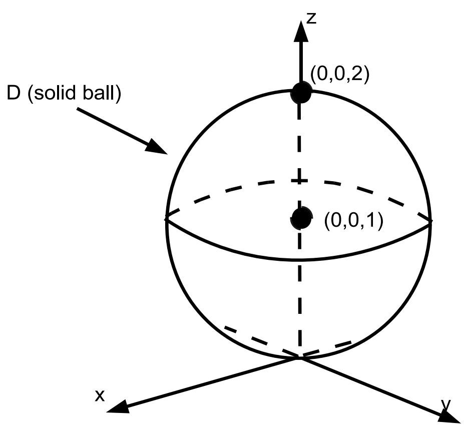
a) Draw the region of integration.
b) Evaluate I.
c) Set up but **do not evaluate** I in the order $dxdy$.
d) Set up but **do not evaluate** I in polar coordinates, in the order $drd\theta$.
e) Set up but **do not evaluate** I in polar coordinates, in the order $d\theta dr$.

10. (7 pts each part, 28 pts total) In this problem, D is the solid ball with center $(0,0,1)$ and radius 1. The equations for the boundary of D in various coordinate systems are:

$$x^2 + y^2 + (z-1)^2 = 1, \qquad r^2 + (z-1)^2 = 1, \qquad \rho = 2\cos\varphi.$$

The density of D is $\delta(x,y,z) = \sqrt{x^2+y^2+z^2}$. We wish to compute the total mass of D.

The rest of problem 10 is on page iv.



Problem 10, continued.

a) Set up **but do not evaluate** an integral giving the total mass of D in **spherical** coordinates.

b) Evaluate the integral from part a).

c) Do the same as part a) (without evaluating!) in **cylindrical** coordinates.

d) Do the same as part a) (no evaluating!) in **rectangular** (i.e., xyz) coordinates.

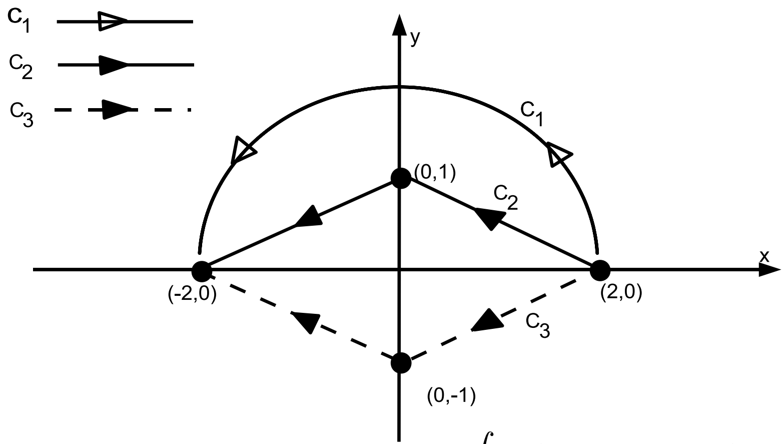
11. (7 pts each part, 14 pts total) Consider the two vector fields in the plane

$$\vec{F} = (0, x), \qquad \vec{G} = (0, y).$$

a) One of the two vector fields is conservative. Explain why, and find a potential function for the conservative field.

b) Let C be the cardioid, given in polar coordinates by $r = 1 + \cos\theta$. We orient C counterclockwise. Find the work integrals $\oint_C \vec{F} \cdot d\vec{r}$ and $\oint_C \vec{G} \cdot d\vec{r}$.

12. (6 pts each part, 18 pts total) The curves C_1, C_2, and C_3 are shown in the figure below. They all go from $(2, 0)$ to $(-2, 0)$.

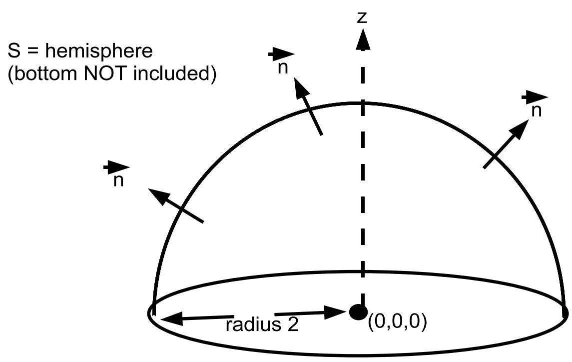


a) Compute the work integral $\int_{C_1} \vec{F} \cdot d\vec{r}$, where $\vec{F} = \dfrac{(-y, x)}{x^2 + y^2}$.

b) Do the same for the integral along C_2. (Hint: what is the curl of $\vec{F}$?)

c) Do the same for C_3, and use this to show that $\vec{F}$ is **not** conservative.

13. (10 pts) Given the vector field in space $\vec{F} = (x, y, z)$. Let S be the upper hemisphere of center $(0, 0, 0)$ and radius 2, oriented with a normal vector that points away from the origin. Compute the flux $\iint_S \vec{F} \cdot \vec{n}\, d\sigma$ of $\vec{F}$ across S.



1. (5 pts each part, 15 pts total) Determine whether each of the following series converges or diverges:

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2-1}}{\sqrt{n^5+1}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \tan^{-1} n$ (c) $\displaystyle\sum_{n=1}^{\infty} (n+1)(0.5)^n$

2. (12 pts) Find the interval of convergence of the following series. Remember to test the endpoints.

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a) Find the first four nonzero terms of the Taylor series for e^x and $\cos 3x$ centered at $a = 0$.

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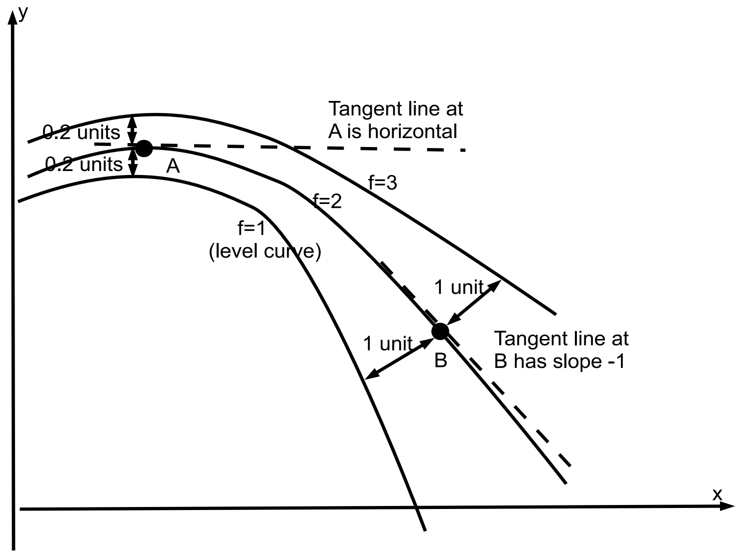

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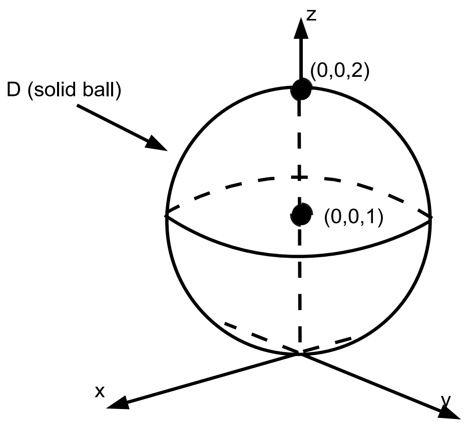

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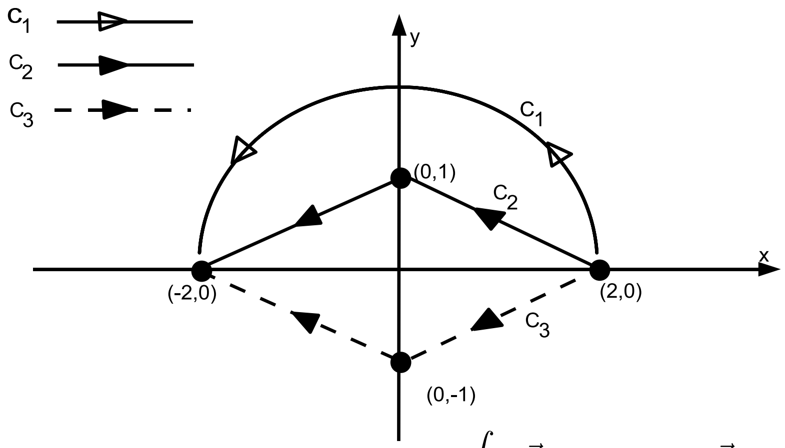

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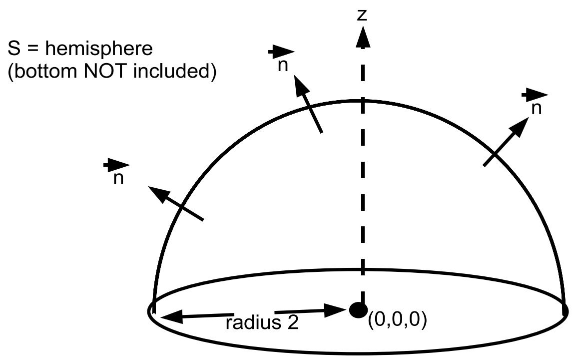

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