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2004 1527

MEN 330
EXAM 2
FALL 2006

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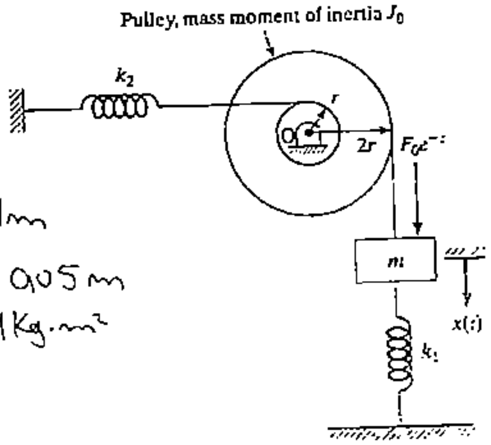
1) (15 pts) For the system shown, let $F(t) = F_0 e^{-t}$, $k_1 = 1000\,N/m$, $k_2 = 500\,N/m$, $r = 5\,cm$, $m = 10\,kg$, $J_0 = 1\,kg \cdot m^2$, and $F_0 = 50\,N$. Set up the equation of motion of the system to be solved by the Laplace transform, then take the Laplace transform of both sides of the equation but do not proceed any further.



$F(t) = F_0 e^{-t}$ $K_1 = 1000\,N/m$
$k_2 = 500\,N/m$ $r = 0.05\,m$
$m = 10\,kg$ $J_0 = 1\,Kg\cdot m^2$
$F_0 = 50\,N$.

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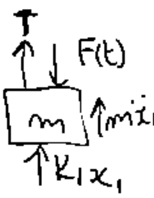


equation of motion for mass "m":

$+T - F(t) + m\ddot{x}_1 + K_1x_1 = 0.$

$T = F(t) - m\ddot{x}_1 - K_1x_1$

equation of motion for the pulley:

$\Sigma M_O = J_0\ddot{\theta}$

$\Rightarrow T(2r) - K_2x_2r = J_0\ddot{\theta}$ $\qquad x_1 = 2r\theta \quad x_2 = r\theta$

$\Rightarrow [F_0e^{-t} - K_1(2r\theta) - m(2r\ddot{\theta})](2r) - K_2r^2\theta = J_0\ddot{\theta}$

$2rF_0e^{-t} - K_14r^2\theta - 4r^2m\ddot{\theta} - K_2r^2\theta = J_0\ddot{\theta}$

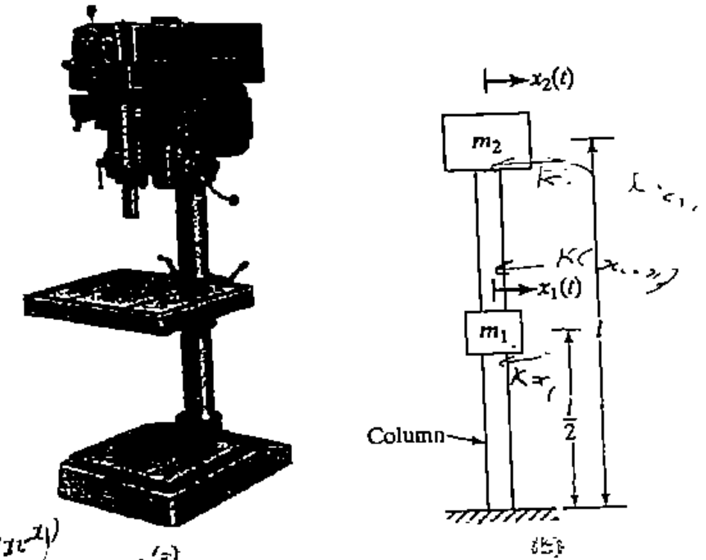
$\ddot{\theta}(J_0 + 4r^2m) + \theta(4r^2K_1 + K_2r^2) = 2rF_0e^{-t}.$

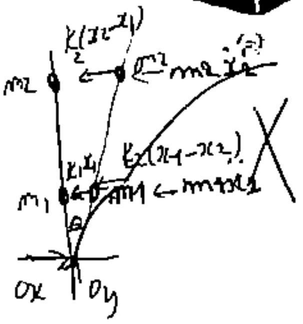
$1{,}1\ddot{\theta} + 11{,}25\theta = 5e^{-t}.$ ✓

Take laplace transform to both sides:

$(1{,}1s^2 + 11{,}25)\,\theta(s) = 5\left(\dfrac{1}{s+1}\right) \quad \Rightarrow (1{,}1s^2 + 11{,}25)\,\theta(s) = \dfrac{5}{s+1}.$ ✓

2) (15 pts) The drilling machine, shown in part (a) of the figure, is modeled as a two-degree-of-freedom system, as shown in part (b) of the figure. The lower and upper columns have stiffnesses k_1 and k_2, respectively. For small horizontal vibrations, draw a free-body diagram of the machine and set up the equations of motion in matrix form. You do not need to solve the system.





equation of motion:

For mass m1:

$$m_1\ddot{x}_1 + K_1x_1 + K_2(x_1-x_2) = 0.$$

$$m_1\ddot{x}_1 + K_1x_1 + K_2x_1 - K_2x_2 = 0.$$

$$m_1\ddot{x}_1 + x_1(K_1+K_2) - K_2x_2 = 0.$$

For mass m2:

$$m_2\ddot{x}_2 + K_2(x_2-x_1) = 0$$

$$m_2\ddot{x}_2 - K_2x_1 + K_2x_2 = 0.$$

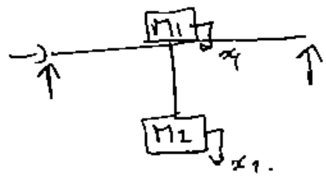
Matrix Form:

$$\begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} K_1+K_2 & -K_2 \\ -K_2 & K_2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

3) (20 pts) A simply-supported steel beam carries a mass $M_1 = 1000\,kg$ at its mid-span. A steel cable from which a mass $M_2 = 5000\,kg$ is suspended, hangs from the middle of the beam. The length of the beam is $L = 40\,m$, its modulus of elasticity is $E = 2.06\times10^{11}\,Pa$ and its moment of inertia is $I = 0.02\,m^4$. The stiffness of the cable is $k_c = 3.0\times10^5\,N/m$. Calculate the natural frequencies of the system for small oscillations.

$m_1 = 1000\,kg \quad m_2 = 5000\,kg \quad L = 40\,m \quad I = 0{,}02\,m^4$

$E = 2{,}06\times10^{11}\,Pa \quad K_c = 3\times10^5\,N/m.$



equation of motion of mass m_1
we consider the simply supported beam to have a stiffness of $K_1 = \frac{48EI}{l^3}$

$m_1\ddot{x}_1 + K_1x_1 + K_c(x_1 - x_2) = 0$

$m_1\ddot{x}_1 + x_1(K_1 + K_c) - K_cx_2 = 0 \quad (1).$

equation of motion for m_2:

$m_2\ddot{x}_2 + K_c(x_2 - x_1) = 0$

$m_2\ddot{x}_2 - K_cx_1 + K_cx_2 = 0 \quad (2)$

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$K_1 = \frac{48\times2{,}06\times10^{11}\times0{,}02}{40^3}$

$K_1 = 3{,}09\times10^6\,N/m$

$K_1 = 3090\,kN/m$

$$\Rightarrow \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} K_1+K_c & -K_c \\ -K_c & K_c \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} 3390 & -300 \\ -300 & 300 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\det\begin{pmatrix} -w^2+3390 & -300 \\ -300 & -5w^2+300 \end{pmatrix} = 0$$

$\Rightarrow 5w^4 - 17250w^2 + 927000 = 0$

$w_1^2 = 54{,}6 \Rightarrow w_1 = 7{,}39\,rad/s$

$w_2^2 = 3395{,}4 \Rightarrow w_2 = 58{,}27\,rad/s$