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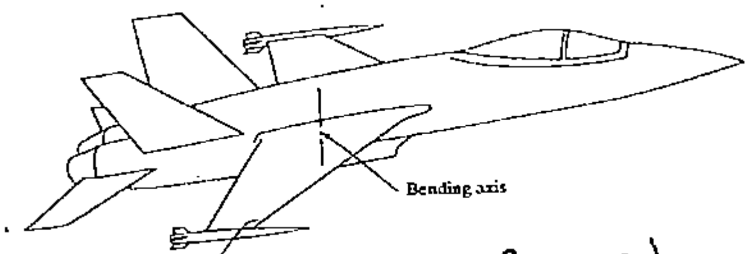
MEN 330
EXAM 2
SPRING 2006

70

1) (15 pts) The wing of a fighter jet carrying a missile at its tip can be approximated as an equivalent cantilever beam with $EI = 15\times10^9\, N\cdot m^2$ about the vertical axis and length $L = 10m$. If the equivalent mass of the wing, including the mass of the missile and its carriage system at the tip of the wing is $M = 2500kg$, determine the vibration response of the wing (i.e., of M) due to the release of the missile. Assume that the force on M due to the release of the missile can be approximated as an impulse function of magnitude $\hat{F} = 50N\cdot s$.

Case of a cantilever beam with end load.



15

$$K_{eq} = \frac{3EI}{l^3} = \frac{3(15\times10^9\ N.m^2)}{10^3} = 45\times10^6\ N.m^2$$

Thus the response of the wing due to that impulse.

$$x(t) = \frac{\hat{F}e^{-\xi\omega_n t}}{m\omega_d}\sin\omega_d t$$

where $\hat{F} = 50\ N.s$ $\quad \omega_n = \sqrt{\frac{K_{eq}}{M}} = \sqrt{\frac{45\times10^6}{2500}} = 134.16$ rd/s

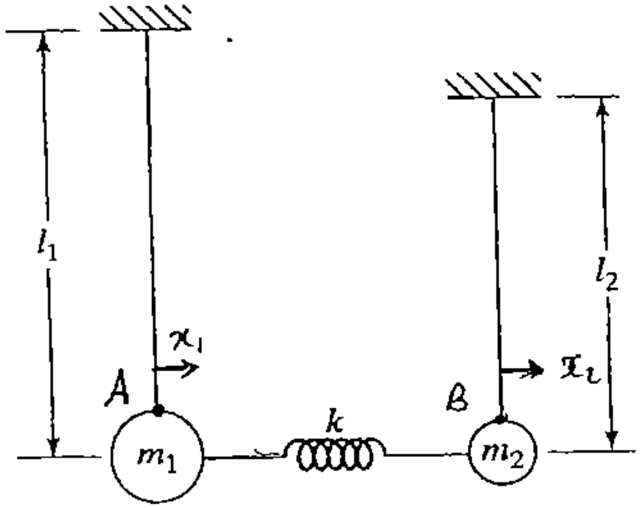
$\omega_d = \omega_n\sqrt{1-\xi^2}$ and $\xi = \frac{c}{2m\omega_n}$.

in case of no damping $\xi = 0$ and $\omega_d = \omega_n$ then the eqn. becomes.

$$\boxed{x(t) = 1.49\times10^{-4}\sin 134.16t}$$

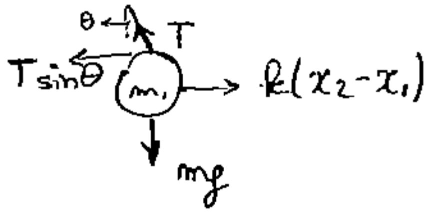
x in meters. t in sec.

2) (15 pts) Assuming small rotational oscillations, set up the equations of motion in matrix form for the double pendulum shown by using two rotational coordinates θ_1 and θ_2.



8

at instant t



where $mg = T\cos\theta$ for both masses

where $x_1 = L_1\theta_1$ and $x_2 = L_2\theta_2$ for small $\angle$'s

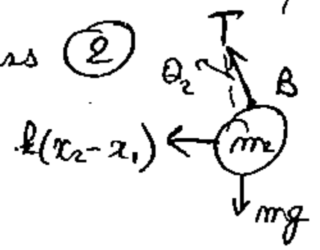
ΣM_A for mass (1) is

$$\Sigma M_A = k(x_2 - x_1)r_1 - (J_1 + m_1 r_1^2)\ddot{\theta}_1 = 0$$

where $L_1\ddot{\theta}_1 = \ddot{x}_1$ and $L_2\ddot{\theta}_2 = \ddot{x}_2$

thus $$\boxed{-(J_1 + m_1 r_1^2)\frac{\ddot{x}_1}{L_1} + k x_2 r_1 - k x_1 r_1 = 0} \quad \ldots (1)$$

for mass (2)



$$\Rightarrow k(x_2 - x_1)r_2 + (J_2 + m_2 r_2^2)\ddot{\theta}_2 = \Sigma M$$

where $\ddot{\theta}_2 = \dfrac{\ddot{x}_2}{L_2}$

$$\Rightarrow \boxed{+(J_2 + m_2 r_2^2)\frac{\ddot{x}_2}{L_2} + k x_2 r_2 - k x_1 r_2 = 0} \quad \ldots (2)$$

The new eqns. of motion become as $x_1 = L_1\theta_1$ and $x_2 = L_2\theta_2$

$$-(J_1 + m_1 r_1^2)\ddot{\theta}_1 + kL_2 r_1 \theta_2 - kL_1 r_1 \theta_1 = 0 \quad \ldots (1)$$

$$+(J_2 + m_2 r_2^2)\ddot{\theta}_2 + kL_2 \theta_2 r_2 - kL_1 r_2 \theta_1 = 0 \ldots (2)$$

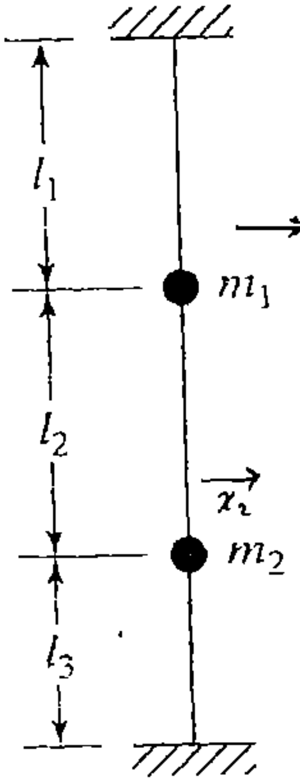
$$\begin{pmatrix} -(J_1 + m_1 r_1^2) & 0 \\ 0 & +(J_2 + m_2 r_2^2) \end{pmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} + \begin{pmatrix} -kL_1 r_1 & kL_2 r_1 \\ -kL_1 r_2 & kL_2 r_2 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

X

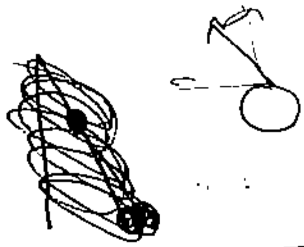
in matrix form.

Assuming that the tension in the massless string is constant and is equal to T, proper free-body diagram for the two masses undergoing small horizontal ons by using translational coordinates x_1 and x_2, and set up the equations of n matrix form.

$= \begin{pmatrix} 0 \\ 0 \end{pmatrix}$



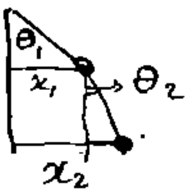
If m_1 is displaced an amount x_1 then m_2 will displace by x_2



for mass ①

$m_1\ddot{x}_1 = 0$

$\boxed{+ T\sin\theta_1 = 0}$ ... ①

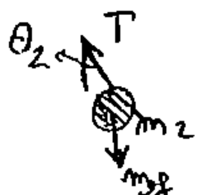


$\boxed{x_2 = x_1 + L_2\theta_2}$

$\Rightarrow \ddot{x}_2 = \ddot{x}_1 + L_2\ddot{\theta}_2$

$x_1 = L_1\theta_1 \quad \Rightarrow \boxed{\theta_1 = \frac{x_1}{L_1}}$

2)



$\sin\theta \simeq \theta$ for small θ

$T\sin\theta_2 \boxed{= 0}$ ... ②

$\theta_2 = \frac{x_2 - x_1}{L_2}$

$\boxed{m_1\ddot{x}_1 + T\frac{x_1}{L_1} = 0}$ and $\boxed{m_2\ddot{x}_2 + T\left(\frac{x_2 - x_1}{L_2}\right) = 0}$

$T x_1 = 0$ X and $m_2 L_2 \ddot{x}_2 + T(x_2 - x_1) = 0$ X