Notre Dame University
Mechanical Engineering Department

MEN 330 MECHANICAL VIBRATIONS

HW#2 – Solution

Pb 2.5

$$m = \frac{9000}{9.8}$$

Let $\omega_n = 7.5$ rad/sec

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

$$k_{eq} = m\omega_n^2 = \left[\frac{9000}{9.8}\right](7.5)^2 = 51658.2 \text{ N/m} = 4\text{ k}$$

where k is the stiffness of the air spring.

$$\text{Thus } k = \frac{51658.2}{4} = 12914.5 \text{ N/m}$$

Pb 2.13

Let $x_1, x_2 =$ **displacement of pulleys 1 and 2**

$x = 2x_1 + 2x_2$ **(1)**

Let P = tension in the rope.

For equilibrium of pulley 1: $2P = k_1 x_1$ **(2)**

For equilibrium of pulley 2: $2P = k_2 x_2$ **(3)**

Where

$$\frac{1}{k_1} = \frac{1}{4k} + \frac{1}{4k} = \frac{1}{2k}, \qquad k_1 = 2k$$

And $k_2 = k + k = 2k$

Combining Eqs (1) to (3)

$$x = 2x_1 + 2x_2 = 2\left(\frac{2P}{k_1}\right) + 2\left(\frac{2P}{k_2}\right) = 4P\left(\frac{1}{2k} + \frac{1}{2k}\right) = \frac{4P}{k}$$

Let $k_{eq} =$ **equivalent spring constant of the system:**

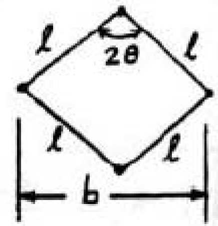
Equation of motion of mass m: $m\ddot{x} + k_{eq}x = 0$

$$\therefore \omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k}{4m}}$$

Pb 2.21

$b = 2\ell \sin\theta$

Neglect masses of links.



(a) $$k_{eq} = k\left(\frac{4\ell^2 - b^2}{b^2}\right) = k\left(\frac{4\ell^2 - 4\ell^2\sin^2\theta}{4\ell^2\sin^2\theta}\right)$$

$$= k\left(\frac{\cos^2\theta}{\sin^2\theta}\right)$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{k\,g\,\operatorname{cosec}^2\theta}{W}}$$

(from solution of problem 1.8)

(b) $\omega_n = \sqrt{\dfrac{k\,g}{W}}$ since $k_{eq} = k$.

Pb 2.33

Assume same area of cross section for all segments of the cable. Speed of blades = 300 rpm = 5 Hz = 31.416 rad/sec.

$$\omega_n^2 = \frac{k_{eq}}{m} = (2\,(31.416))^2 = (62.832)^2$$

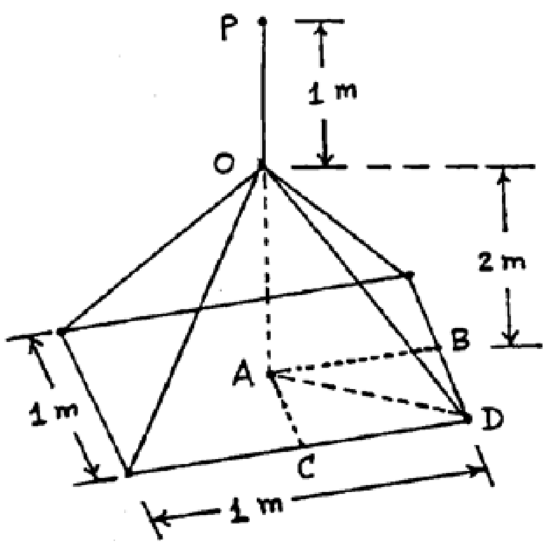
$$k_{eq} = m\,\omega_n^2 = 250\,(62.832)^2 = 98.6965\,(10^4)\text{ N/m} \qquad (1)$$

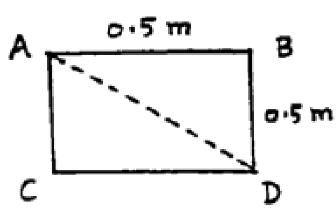
$$AD = \sqrt{0.5^2 + 0.5^2} = 0.7071\text{ m}\;,\quad OD = \sqrt{2^2 + 0.7071^2} = 2.1213\text{ m}$$

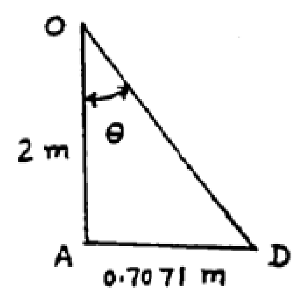
Stiffness of cable segments:

$$k_{PO} = \frac{A\,E}{\ell_{PO}} = \frac{A\,(207)\,(10^9)}{1} = 207\,(10^9)\,A\text{ N/m}$$

$$K_{OD} = \frac{A\,E}{\ell_{OD}} = \frac{A\,(207)\,(10^9)}{2.1213} = 97.5817\,(10^9)\,A\text{ N/m}$$







$$\theta = \tan^{-1}\left(\frac{0.7071}{2}\right) = 19.4710°$$

The total sttiffness of the four inclined cables (k_{ic}) is given by:

$$k_{ic} = 4\, k_{OD} \cos^2\theta$$
$$= 4\,(97.5817)\,(10^9)\,A \cos^2 19.4710° = 346.9581\,(10^9)\,A\ N/m$$

Equivalent stiffness of vertical and inclined cables is given by:

$$\frac{1}{k_{eq}} = \frac{1}{k_{PO}} + \frac{1}{k_{ic}}$$

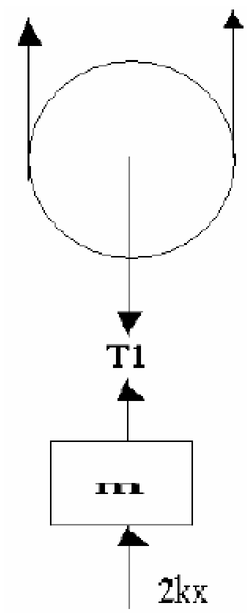
i.e., $$k_{eq} = \frac{k_{PO}\, k_{ic}}{k_{PO} + k_{ic}}$$

$$= \frac{(207\,(10^9)\,A)\,(346.9581\,(10^9)\,A)}{(207\,(10^9)\,A) + (346.9581\,(10^9)\,A)} = 129.6494\,(10^9)\,A\ N/m \qquad (2)$$

Equating k_{eq} given by Eqs. (1) and (2), we obtain the area of cross section of cables as:

$$A = \frac{98.6965\,(10^4)}{129.6494\,(10^9)} = 7.6126\,(10^{-6})\ m^2$$

Pb 2.46



Consider the springs connected to the pulleys (by rope) to be in series. Then:

$$\frac{1}{k_{eq}} = \frac{1}{k} + \frac{1}{5k}$$

$$\therefore k_{eq} = \frac{5}{6}k$$

Let the displacement of mass m be x.

Then the extension of the rope (Spring Connected to the pulleys) = $2x$.

From the free body diagram , the equation of motion m becomes:

$$m\ddot{x} + 2kx + k_{eq} = 0$$

$$\therefore m\ddot{x} + \frac{11}{3}kx = 0$$

Pb 2.48

T= Kinetic energy = $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}J_0\dot{\theta}^2$

U= potential energy = $\frac{1}{2}k_s^2$

Where $\theta = \frac{x}{r}$, x_s = extension of spring = $4r\theta = 4x$

Hence,

$$T=\frac{1}{2}\left(m+\frac{J_0}{r^2}\right)\dot{x}^2 \quad ; \quad U=\frac{1}{2}(16k)x^2$$

Using the relation $\frac{d}{dt}(T+U)=0$, we obtain the equation of motion of the system as:

$$\left(m+\frac{J_0}{r^2}\right)\ddot{x}+16kx=0$$

Pb 2.93

Assume that the bicycle and the boy fall as a rigid body by 5 cm at point A. Thus the mass (m_{eq}) will be subjected to an initial downward displacement of 5 cm (t = 0 assumed at point A):

$$x_0 = 0.05 \text{ m}, \; \dot{x}_0 = 0$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{(50000)\,(9.81)}{800}} = 24.7614 \text{ rad/sec}$$

$$c_c = 2\,m\,\omega_n = 2\left(\frac{800}{9.81}\right)(24.7614) = 4038.5566 \text{ N-s/m}$$

$$\zeta = \frac{c}{c_c} = \frac{1000.0}{4038.5566} = 0.2476 \text{ (underdamping)}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 24.7614\sqrt{1-0.2476^2} = 23.9905 \text{ rad/sec}$$

Response of the system:

$$x(t) = X\,e^{-\zeta\,\omega_n\,t}\sin(\omega_d\,t+\phi)$$

where $X = \left\{x_0^2 + \left(\frac{\dot{x}_0+\zeta\,\omega_n\,x_0}{\omega_d}\right)^2\right\}^{\frac{1}{2}}$

$$= \left\{(0.05)^2 + \left(\frac{(0.2476)\,(24.7614)\,(0.05)}{23.9905}\right)^2\right\}^{\frac{1}{2}} = 0.051607 \text{ m}$$

and $\phi = \tan^{-1}\left(\frac{x_0\,\omega_d}{\dot{x}_0+\zeta\,\omega_n\,x_0}\right) = \tan^{-1}\left(\frac{0.05\,(23.9905)}{0.2476\,(24.7614)\,(0.05)}\right) = 75.6645°$

Thus the displacement of the boy (positive downward) in vertical direction is given by

$$x(t) = 0.051607\,e^{-6.1309\,t}\sin(23.9905\,t + 75.6645°) \text{ m}$$