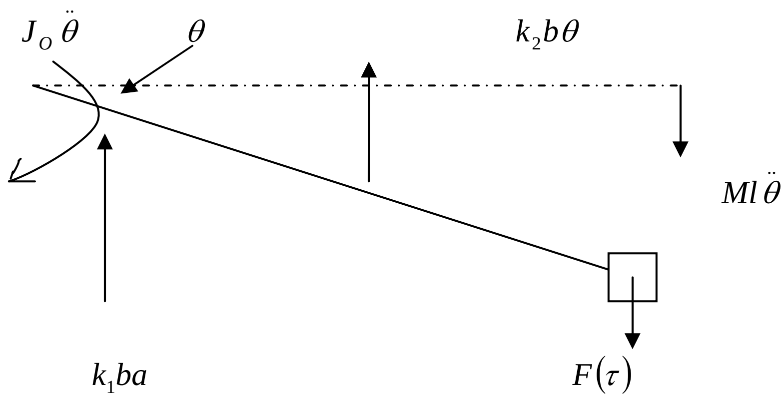
Notre Dame University
Mechanical Engineering Department

MEN 330 MECHANICAL VIBRATIONS

HW#3 – Solution

Pb 3.19



E.O.M. for the relation around hinge O

$$\left(J_O + Ml^2\right)\ddot{\theta} + \left(k_1 a^2 + k_2 b^2\right)\theta = F(t)l$$
$$= F_O l \sin wt$$

Response of an undamped system under harmonic motion

$$\theta_p(t) = \theta \sin wt$$

where,

$$\theta = \frac{F_O l}{\left(k_1 a^2 + k_2 b^2\right) - \left(J_O + Ml^2\right)w^2} \qquad \text{(similar to equation 3.3)}$$

and

$$J_O = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 = \frac{1}{3}ml^2$$

$*$ For the given data, $J_O = \frac{1}{3}(10)\left(1^2\right) = 3.333 \ kg.m^2$

$$w = \frac{1000(2\Pi)}{60} = 104.72 \ \ rad/\sec$$

and

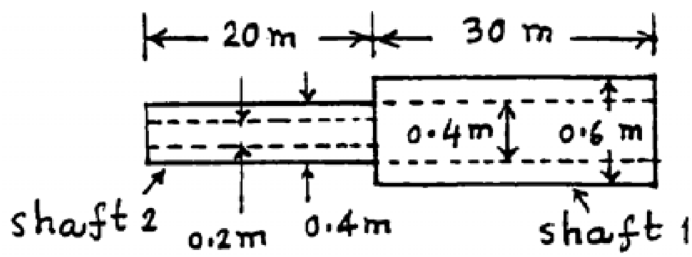
$$\theta = \left| \frac{500(1)}{5000(0.25^2 + 0.5^5) - (3.333 + 50(1)^2)(104.72^2)} \right| = 0.5718 \times 10^{-4} \ \ rad$$

Pb 3.26

Equation of motion for torsional system:

$$J_0 \ddot{\theta} + c_t (\dot{\theta} - \dot{\alpha}) + k_t (\theta - \alpha) = 0 \tag{1}$$

where $\theta =$ angular displacement of shaft and $\alpha =$ angular displacement of base of shaft $= \alpha_0 \sin \omega t$. Steady state response of propeller (Eq. (3.67)):



$$\theta_p(t) = \Theta \sin (\omega t - \phi) \tag{2}$$

$$\text{where } \Theta = \alpha_0 \left\{ \frac{k_t^2 + (c_t \omega)^2}{(k_t - J_0 \omega^2)^2 - (c_t \omega)^2} \right\}^{\frac{1}{2}} \tag{3}$$

$$\text{and } \phi = \tan^{-1} \left\{ \frac{J_0 c_t \omega^3}{k_t (k_t - J_0 \omega^2) + (c_t \omega)^2} \right\} \tag{4}$$

Here $J_0 = 10^4$ kg–m², $\zeta_t = 0.1$, and $\omega = 314.16$ rad/sec. Torsional stiffnesses of shafts:

$$(k_t)_1 = \frac{G_1 J_1}{\ell_1} = \frac{(80 (10^9)) \left(\frac{\pi}{32} (0.6^4 - 0.4^4) \right)}{30} = 27.2272 (10^6) \text{ N–m/rad}$$

$$(k_t)_2 = \frac{G_2 J_2}{\ell_2} = \frac{(80 (10^9)) \left(\frac{\pi}{32} (0.4^4 - 0.2^4) \right)}{20} = 9.4248 (10^6) \text{ N–m/rad}$$

Series springs give:

$$k_t = \frac{(k_t)_1\ (k_t)_2}{(k_t)_1 + (k_t)_2} = \frac{(27.2272\ (10^6))\ (9.4248\ (10^6))}{27.2272\ (10^6) + 9.4248\ (10^6)} = 7.0013\ (10^6)\ \text{N—m/rad}$$

$$c_t = \varsigma\ (2\sqrt{J_0\ k_t}) = 0.1\ (2)\sqrt{(10^4)\ (7.0013\ (10^6))} = 52{,}919.8624\ \text{N—m—s/rad}$$

From Eq. (3),

$$\Theta = 0.05\left[\frac{(7.0013\ (10^6))^2 + \left\{5.2920\ (10^4)\ (314.16^2)\right\}^2}{\left\{7.0013\ (10^6) - (10^4)\ (314.16^2)\right\}^2 + \left\{5.2920\ (10^4)\ (314.16)\right\}^2}\right]^{\frac{1}{2}}$$

$$= 9.2028\ (10^{-4})\ \text{rad}$$

$$\phi = \tan^{-1}\left\{\frac{(10^4)\ (5.2920\ (10^4))\ (314.16^3)}{7.0013\ (10^6)\left[7.0013\ (10^6) - (10^4)\ (314.16^2)\right] + (5.2920\ (10^4)\ (314.16))^2}\right\}$$

$$= \tan^{-1}\ (59.3664) = 89.0350^\circ = 1.5540\ \text{rad}$$

Pb 3.36

$$\frac{X}{Y} = \left[\frac{k^2 + c^2\ \omega^2}{\left(k - m\ \omega^2\right)^2 + c^2\ \omega^2}\right]^{\frac{1}{2}}$$ .sp

$$\text{or}\quad \frac{10^{-6}}{Y} = \left[\frac{(10^6) + (10^3\ (200\ \pi))^2}{\left\{10^6 - \left(\frac{5000}{9.81}\right)(200\ \pi)^2\right\}^2 + \left\{(10^3)\ (200\ \pi)\right\}^2}\right]^{\frac{1}{2}}$$

$$\text{or}\quad Y = 169.5294\ (10^{-6})\ \text{m}$$

Pb 3.38

$$w = (3000 rpm) = 31.416 \ {rad}/{\sec}$$

Eqtns (3.33) and (3.34) yield

$$w = w_n \sqrt{1-2\zeta^2} = \sqrt{\frac{k}{m}} \sqrt{1-2\zeta^2} = 31.416$$

$$\Rightarrow k(1-2\zeta^2) = (31.416)^2 (100) = 98{,}696.505 \qquad (1)$$

and

$$X_{\max} = \delta_{st} \frac{1}{2\zeta\sqrt{1-\zeta^2}} = \frac{F_O}{k} \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 0.005$$

or $$k\zeta\sqrt{1-\zeta^2} = \frac{F_O}{2(0.005)} = 10{,}000 \qquad (2)$$

Now solve the system of 2 equations, 2 unknowns
Divide (1) by (2):

$$\frac{1-2\zeta^2}{\zeta\sqrt{1-\zeta^2}} = 9.8696$$

Squaring and rearranging leads to:

$$101.4090\zeta^4 - 101.4090\zeta^2 + 1 = 0$$

$$\Rightarrow \zeta = 0.0998 \qquad \text{or} \qquad \zeta = 0.995$$

$*$ using $\zeta = 0.0998$:

$$\Rightarrow k = \frac{98696.5}{1-2(0.0988)^2} = 100{,}702.5 \ {N}/{m}$$

Since $\zeta = \frac{C}{C_c} = \frac{C}{2mw_n}$, we find:

$$C = 2mw_n\zeta = 200\sqrt{\frac{100702.5}{1000}}(0.0998) = 633.4038 \ {N.s}/{m}$$

Pb 3.86

Unbalanced force n vertical direction $= mew^2 \sin wt$
Unbalanced force in horizontal direction = 0

Let M = total mass of the shaker

E.O.M. $M \ddot{x} + c\dot{x} + kx = mew^2 \sin wt$

Steady-state solution:

$$x(t) = X \sin(wt - \phi) \tag{1}$$

where:

$$X = \frac{mer^2}{M\left[\left(1-r^2\right)^2 + \left(2\xi r\right)^2\right]^{1/2}}$$

and

$$\phi = \tan^{-1}\left(\frac{2\xi r}{1-r^2}\right)$$

- Frequency range: 20 < f < 30 Hz

$$\Rightarrow 125.66 < w < 188.5 \ rad/s \tag{2}$$

$$0.1'' \le X \le 0.2in \tag{3}$$

- Mean power output over a time period τ is:

$$P = \frac{1}{\tau}\int_0^\tau F(\tau)\frac{dx}{dt}(\tau)d\tau$$

where $\tau = \frac{2\Pi}{w}$

$$F(\tau) = mew^2 \sin wt$$

$$\frac{dx}{dt} = wX\cos(wt - \phi)$$

$$P \ge 1hp \tag{4}$$

$$\frac{M}{m} \ge 50 \tag{5}$$

Find w, e, M, m, k, and c
So as to satisfy the requirements stated in (2), (3), (4) and (5).