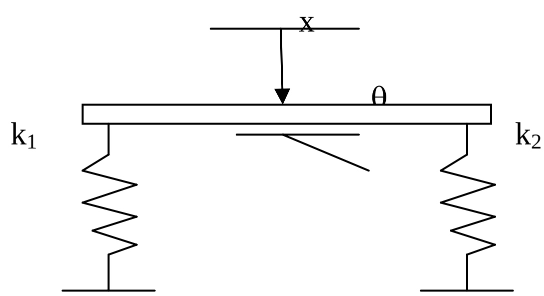
Notre Dame University
Mechanical Engineering Department

MEN 330 MECHANICAL VIBRATIONS

HW#4 – Solution

Pb 5.4



Equations of motion in terms of x and θ

$$m\ddot{x} + k_1(x - l_1\theta) + k_2(x + l_2\theta) = 0$$
$$J_0\ddot{\theta} - k_1 l_1(x - l_1\theta) + k_2 l_2(x + l_2\theta) = 0$$

For free vibrations, assume a solution

$$x(t) = X\cos(\omega t + \emptyset)$$
$$\theta(t) = \Theta\cos(\omega t + \phi)$$

In matrix form,

$$\begin{bmatrix} -m\omega^2 + k_1 + k_2 & -(k_1 l_1 - k_2 l_2) \\ -(k_1 l_1 - k_2 k_2) & -J_o\omega^2 + k_1 l_1^2 + k_2 l_2^2 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is:

$$\begin{vmatrix} -\omega^2 + 5000 & 100 \\ 100 & -0.3\omega^2 + 2030 \end{vmatrix} = 0$$

$\Rightarrow 0.3\omega^4 - 3530\omega^2 + 10.14 \times 10^6 = 0$

ω^2=6785.3373, 4981.3293

$\Rightarrow \omega_1 = 70.5785\, {}^{rad}/_{sec}$ $\omega_2 = 82.3732\, {}^{rad}/_{sec}$

Mode shapes:

$$(-1000\omega_1^2 + 5 \times 10^6)X + 0.1 \times 10^6\Theta = 0$$

Or

$$\left.\frac{X}{\Theta}\right|_{\omega_1} = \frac{-0.1 \times 10^6}{-1000\omega_1^2 + 5 \times 10^6} = -5.3476$$

And

$$\left.\frac{X}{\Theta}\right|_{\omega_2} = \frac{-0.1 \times 10^6}{-1000\omega_2^2 + 5 \times 10^6} = 0.05601$$

Pb 5.7

Frequency Equation:

$$|-\omega^2[m]+[k]|=0$$

$$\begin{vmatrix} k_{11}-\omega^2 m_1 & k_{12} \\ k_{21} & k_{22}-\omega^2 m_2 \end{vmatrix}=0$$

Then,

$$(m_1m_2)\omega^4-(m_1k_{22}+m_2k_{11})\omega^2+(k_{11}k_{22}-k_{12}^2)=0$$

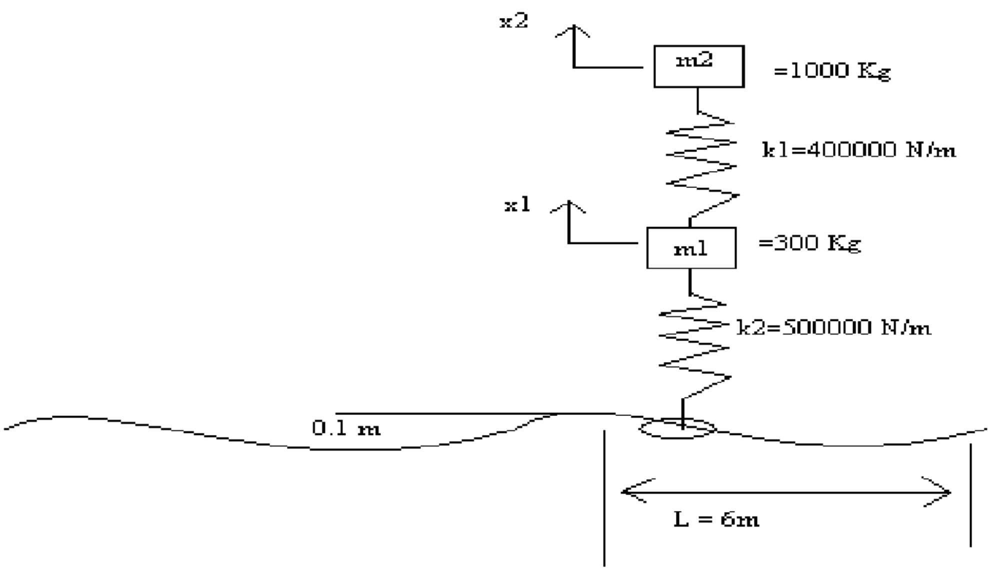
Roots:

$$\omega_{1,2}^2=\frac{m_1k_{22}+m_2k_{11}\mp\sqrt{(m_{11}k_{22}-m_2k_{11})^2+4m_1m_2k_{12}^2}}{2m_1m_2}$$

Substituting known expressions for k_{11}, k_{12} and k_{22} yields:

$$\omega_{1,2}^2=\frac{48}{7}\frac{EI}{m_1m_2}\left[(m_1+8m_2)\mp\sqrt{(m_1-8m_2)^2+25m_1m_2}\right]$$

Pb 5.8



Equations of motion:

$$m_1\ddot{x}_1+k_1x_1-k_1x_2=0 \qquad (E_1)$$

$$m_2\ddot{x}_2+(k_1+k_2)x_2-k_1x_1=0 \qquad (E_2)$$

Let: $x_i(t)=X_i\cos(\omega t+\emptyset)$;
i=1,2

Eq. (E_1) becomes:

$$\begin{bmatrix} -m_1\omega^2+k_1 & -k_1 \\ -k_1 & -m_2\omega^2+k_1+k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is:

$$\begin{vmatrix} -m_1\omega^2+k_1 & -k_1 \\ -k_1 & -m_2\omega^2+k_1+k_2 \end{vmatrix}=0$$

i.e.,

$$m_1 m_2 \omega^4 - (m_1 k_1 + m_1 k_2)\omega^2 - k_1 m_2 \omega^2 + k_1 k_2 = 0$$

i.e.,

$$\omega^2 = \left[(m_1 k_1 + m_1 k_2 + k_1 m_2) \mp (m_1^2 k_1^2 + m_1^2 k_2^2 + m_2^2 k_1^2 + 2m_1^2 k_1 k_2 - 2m_1 m_2 k_1 k_2 + 2m_1 m_2 k_1^2)^{\frac{1}{2}}\right] \Big/ 2m_1 m$$

(E3)

Since m_1=1000 Kg, m_2= 300 Kg, k_1= 4x10^5 N/m and k_2=5x10^5 N/m,

Eq. (E3) gives

ω_1=14.4539 rad/sec, ω_2=56.4897 rad/sec

$$f_1 = \frac{14.4539}{2\pi} Hz = \frac{s_1(1000)}{3600}\left(\frac{1}{l}\right) = \frac{s_1}{21.6}$$

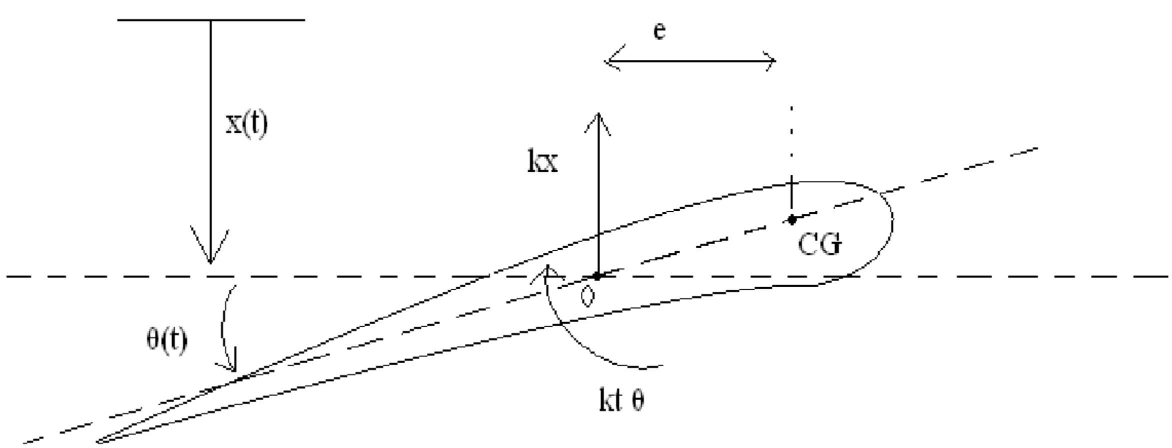
Where l =6m and s_1 is in km/hr

$\Rightarrow s_1$= critical velocity#1 = $\frac{14.4539}{2\pi}(21.6) = 49.6887\, {}^{km}/_{hr}$

$$f_2 = \frac{56.4897}{2\pi} Hz = \frac{s_2(1000)}{3600}\left(\frac{1}{l}\right) = \frac{s_2}{21.6}$$

$\Rightarrow s_1$= critical velocity#2 = $\frac{56.4897}{2\pi}(21.6) = 194.1968\, {}^{km}/_{hr}$

Pb 5.35



Equations of motion:

$$m(\ddot{x} - e\ddot{\theta}) = -kx$$

$$J_{CG}\ddot{\theta} = -k_t\theta - kxe$$

$$\Rightarrow m\ddot{x} + kx - me\ddot{\theta} = 0$$

$$(J_0 - me^2)\ddot{\theta} + k_t\theta + kex = 0$$

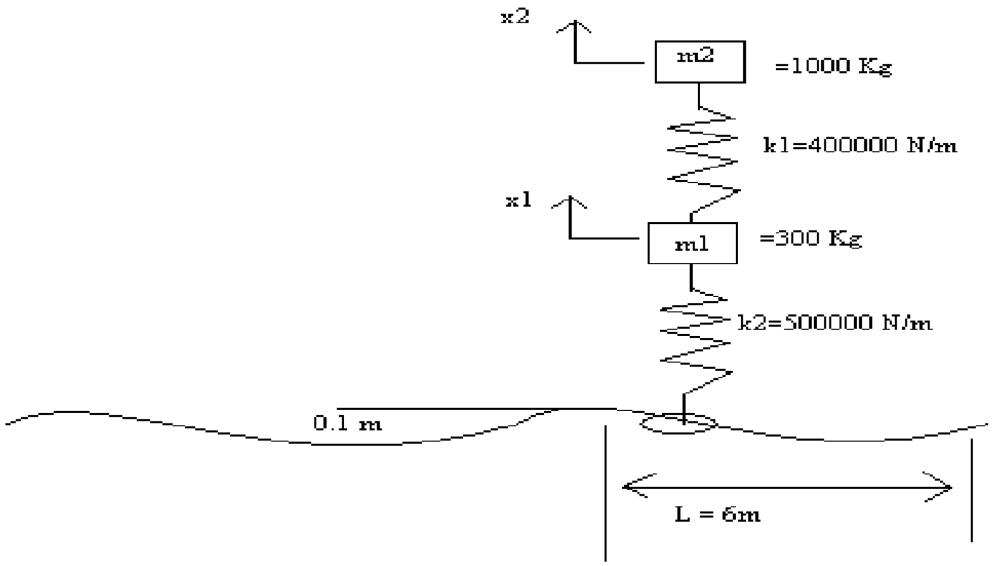
Assuming a harmonic solution, we get the frequency equation as:

$$\begin{vmatrix} -m\omega^2 + k & me\omega^2 \\ ke & -(J_0 - me^2)\omega^2 + k_t \end{vmatrix} = 0$$

Or $\quad (J_0 - me^2)m\omega^4 - (J_0 k + mk_t)\omega^2 + kk_t = 0$

Roots of this equation give the natural frequency of the system.

Pb 5.49



$$k_1 = k_{beam} = \frac{192E\left(\frac{1}{12}at^3\right)}{l^3} = \frac{16Eat^3}{l^3}$$

Equations of motion:

$$\left.\begin{aligned} m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = F_1(t) = F_0 cos\omega t \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) = 0 \end{aligned}\right\} \quad \ldots\ldots\ldots(E_1)$$

Assuming harmonic response

$x_j(t) = X_j cos\omega t \qquad ; \qquad$ j=1,2

Equations (E_1) yield

$$X_1 = \frac{(k_2 - m_2\omega^2)F_0}{(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}$$

$$X_2 = \frac{(k_2)F_0}{(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}$$

For no steady state vibration of the beam, X_1=0 and hence the condition to be satisfied is:

$$\frac{k_2}{m_2} = \omega^2$$