NOTRE DAME UNIVERSITY
Faculty of Natural and Applied Sciences

Department of Mathematics and Statistics

Exam II
STA 207

Date: Thursday August 6th, 2009

Time: 2:00 – 3:55 pm
Duration: 55 minutes

Number of Questions: 5
Number of pages: 8

Exam Policy

Note: Cheating will not be tolerated

- **Put your ID on the desk**
- **Do not keep your cellular on the desk**
- **Look at your exam paper only**
- **Do not talk to others**
- **Do not lend or lent things**
- **Questions are not allowed**

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Question 1: (16 points)

In a study conducted to check the effectiveness of certain exercises on weight reduction, a group of 10 persons were selected and their weights (in pounds), before and after engaging in these exercises over a one month period, were checked and recorded in the following table:.

Weight Before	180	171	172	182	161	183	155	201	203	146
Weight After	174	172	166	180	161	182	153	203	201	142
D_i (Before - After)	6	−1	6	2	0	1	2	−2	2	4

At $\alpha = 0.05$, is there sufficient evidence to believe that the exercises are effective in weight reduction?

we assume the population is normally distributed, samp is small and ~~dependant~~ . so we use t-distribut to test the claim. with $df = n - 1 = 10 - 1 = 9$

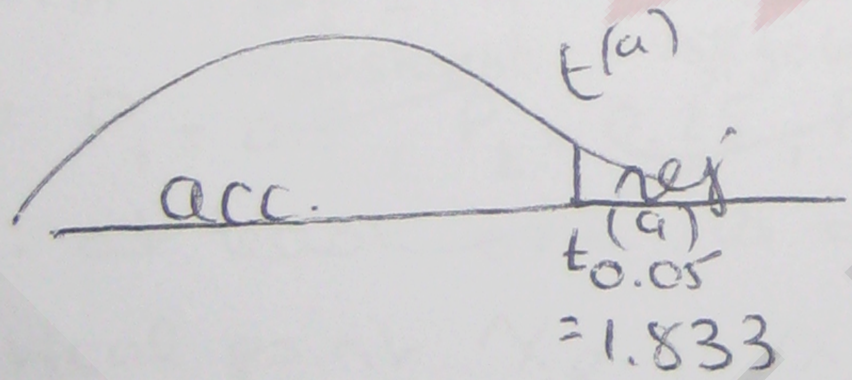
$H_0: \mu_{\bar{D}} = 0$ $\quad H_a: \mu_D > 0$

$\bar{D} = \frac{\sum D_i}{n} = 2$

$S_D = 2.71$

critical point: $X_c = t_\alpha = t_{0.05}^{(9)} = 1.833$

$OUTS = t = \frac{\bar{D} - \mu_D}{S_D/\sqrt{n}} = \frac{2 - 0}{2.71/\sqrt{10}} = 2.33$



OUTS is in the rejection reg so we reject H_0 and we accept the claim.

There is sufficient evidence to believe that the exercises reduce the weight.

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Question 2: (16 points)

When a vehicle arrives at an intersection, it can turn right, turn left, or continue straight ahead. It is claimed that 50% of vehicles entering a specific intersection will continue straight ahead, and of the remaining vehicles, half turn right and half turn left. The following table reveals the direction of 108 vehicles entering the specified intersection:

Direction	Straight	Left	Right
Frequency	59	29	20

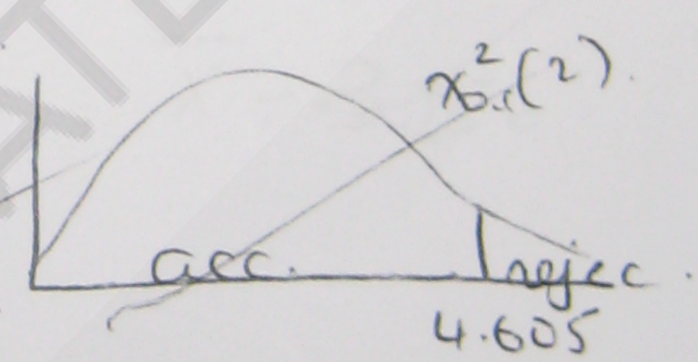
At $\alpha = 0.10$, is there sufficient evidence to reject the claim?

Direction	frequency (Observed)	Expected	O - E
straight	59	50% of 108 = 54	5
left	29	25% of 108 = 27	2
right	20	25% of 108 = 27	-7

We assume that the ~~population is normally~~ distributed. and the sample is randomly selected. so to test the claim of goodness of fit we use χ^2 distribution.

with $df = k - 1 = 3 - 1 = 2$

H_0: $P_1 = 0.5$ (50% go straight), $P_2 = 0.25$ (25% go left), $P_3 = 0.25$ (25% go right)



H_a: at least one is different.

critical point $\chi_R = \chi^2_{0.1}(2) = 4.605$

$OVTS = \chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}} = 0.463 + 0.148 + 1.815 = 2.426$

since OVTS is in the acceptance region, then we accept Ho and the claim.

So no there isn't enough evidence to reject the claim

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Question 3: (29 points)

Three independent random samples were selected from three normally distributed populations with common but unknown variance σ^2. The data are shown below

Sample (1)	3.1	4.3	2.2	4.3	4	
Sample (2)	5.4	3.6	4.0	2.9	2.2	3.6
Sample (3)	2.6	2.2	3.0	2.7	2	

At $\alpha = 0.05$, is there sufficient evidence to conclude that all the true means of the three populations are equal? Use the P-value method.

~~because~~ since the populations are normally distributed and the samples are randomly selected, ~~and we assume they have equal~~ then we use ~~f~~ the ANOVA method with the f distribution.

$H_0: \mu_1 = \mu_2 = \mu_3$

H_a: at least one is different.

$df_1 : (K-1) = 3-1 = 2$

$df_2 : (N-K) = 16-3 = 13$

$\bar{X}_1 = 3.58$ $\quad \bar{X}_2 = 3.62$ $\quad \bar{X}_3 = 2.5$ $\quad \bar{X} = 3.256$

$S_1^2 = 0.837$ $\quad S_2^2 = 1.17$ $\quad S_3^2 = 0.16$

$n_1 = 5$ $\quad n_2 = 6$ $\quad n_3 = 5$

$SSB = n_i(\bar{x}_i - \bar{x})^2 = 5(3.58-3.256)^2 + 6(3.62-3.256)^2 + 5(2.5-3.256)^2$
$= 0.525 + 0.795 + 2.858 = 4.178$

$SSW = (n_i - 1)S_i^2 = (5-1)0.837 + 5(1.17) + 4(0.16)$
$= 3.348 + 5.85 + 0.64 = 9.838$

$MSB = \frac{SSB}{K-1} = \frac{4.178}{2} = 2.089$

$MSW = \frac{SSW}{N-K} = \frac{9.838}{13} = 0.757$

$OUTS = f = \frac{MSB}{MSW} = \frac{2.089}{0.757} = 2.759$

Pvalue $= P(F_{(2,13)} > OUTS)$
$= P(F > 2.759)$

$\alpha = 0.1 \to 2.76317$
$\alpha = 0.05 \to 3.8056$
$\alpha = 0.025 \to 4.9653$
$\alpha = 0.01 \to 6.701$

Pvalue $= 0.1$

Since Pvalue $> \alpha$ then we accept H_0 and we accept the claim that there is sufficient evidence that all ~~the~~ means are equal.

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Question 4: (21 Points)

The strength of concrete depends, to some extent, on the method used for drying. Two different drying methods were applied to two independent random samples of concrete. The results are a shown below:

Method I	Method II
$n_1 = 8$	$n_2 = 13$
$\bar{x}_1 = 3250$	$\bar{x}_2 = 3150$
$s_1 = 210$	$s_2 = 175$

Construct a 90% confidence interval for $\mu_1 - \mu_2$ assuming that the two populations from which our samples were selected are normal with:

1 (b) different variances

~~(a) equal variances~~ (12 points)

Assume that 2 populations are normal and $\sigma_1^2 \neq \sigma_2^2$ then confidence interval $\mu_1 - \mu_2$ used is

$$\left[(\bar{X}_1 - \bar{X}_2) - Z_{\alpha/2} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \, , \, (\bar{X}_1 - \bar{X}_2) + Z_{\alpha/2} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \right]$$

t

90% → $1 - \alpha = 0.9$, $\alpha = 0.1$

$Z_{\alpha/2} = Z_{0.1/2} = Z_{0.05} = 1.645$

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$$\left[(3250 - 3150) - 1.645 \sqrt{\frac{210^2}{8} + \frac{175^2}{13}} \, , \, (3250 - 3150) + 1.645 \sqrt{\frac{210^2}{8} + \frac{17}{1}} \right.$$

$$\left[(100) - 145.92 \, , \, 100 + 145.92 \right]$$

$$\left[-45.92 \, , \, 245.92 \right]$$

we are 90% confident that $\mu_1 - \mu_2$ is between [-45.92 and 245.92.

equal

b) ~~different~~ variances. (9 points) why Z? $s_1^2 = s_2^2$

$$(\bar{X}_1 - \bar{X}_2) - \overset{t}{Z_{\alpha/2}} \underbrace{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2}}}_{}{}^{S_p} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}, \dots + \dots$$

$$(3250 - 3150) - 1.645 \sqrt{\frac{(8-1)210^2 + (13-1)175^2}{8+13-2}} \times \sqrt{\frac{1}{8} + \frac{1}{13}}$$

$$[100 - 1.645\,(188.65)(0.449),\ 100 + \dots]$$

$$[100 - 139.33,\ 100 + 139.33]$$

$$[-39.33,\ 239.33]$$

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we are 90% conf that the interval is between -39.33 and 239.33

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Question 5: (18 points)

To study the proportions of people who suffer from lung diseases in two separate industrial areas, the researchers found that 57 out of 200 people selected from area (1) and 30 out of 125 people selected from area (2) suffer from lung diseases. At $\alpha = 0.05$, can we conclude that there is a difference in the true proportions of people who suffer from lung diseases in both areas?

(1) 57 of 200 $\rightarrow \hat{P}_1 = 0.285$ $\quad \hat{q}_1 = 0.715$

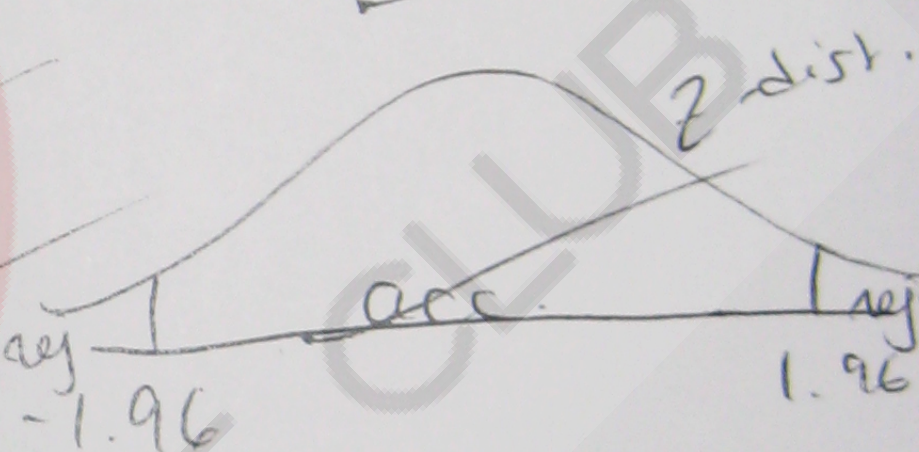
(2) 30 of 125 $\rightarrow \hat{P}_2 = 0.24$ $\quad \hat{q}_2 = 0.76$

$H_0: P_1 - P_2 = 0$ $\qquad H_a: P_1 - P_2 \neq 0$

Since we assume the ~~population is normal~~ and ~~the samples~~ size of both samples are large then we use Z-distribution to ~~do~~ test the claim.

$X_L = -Z_{\alpha/2} = -Z_{0.05/2} = -1.96$

$X_R = +Z_{\alpha/2} = Z_{0.05/2} = 1.96$



OUTS = ~~$(\hat{P}_1 - \hat{P}_2)$~~ $Z = \dfrac{\hat{P}_1 - \hat{P}_2 - (P_1 - P_2)}{\sqrt{P_0 q_0 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$

$P_0 = \dfrac{n_1 \hat{P}_1 + n_2 \hat{P}_2}{n_1 + n_2} = \dfrac{57 + 30}{325} = 0.268$

$q_0 = 1 - 0.268 = 0.732$

$\text{OUTS} = \dfrac{(0.285 - 0.24) - (0)}{\sqrt{(0.268)(0.732)\left(\frac{1}{200} + \frac{1}{125}\right)}} = \dfrac{0.045}{0.0505} = 0.891$

Since OUTS is in the acceptance ~~region~~.
So we accept H_0 and we reject the claim
there isn't a difference in ~~the~~ true proportions.