MATH201 Prerequisites

Course textbook: Finney, Weir, and Giordano, *Thomas' Calculus*, 11th edition.

Below are some main points you should be familiar with. If you feel you need to practice more about a few topics you should check the textbook. If you feel that you need to learn most of these topics you should take MATH102! These topics are covered in the following sections of the textbook: 1.6, 7.1, 7.2, 7.3, 7.4, 7.7, 7.8, 12.1–12.5, 8.1–8.5.

Inverse Functions and Logarithms

- One-to-one and Inverse Functions
- Logarithmic functions
  $y = \log_a x \Leftrightarrow x = a^y$, $\log_{10} x = \log x$, $\log_e x = \ln x$

$$(0,\infty) \underset{a^x}{\overset{\log_a x}{\rightleftarrows}} (-\infty,\infty)$$

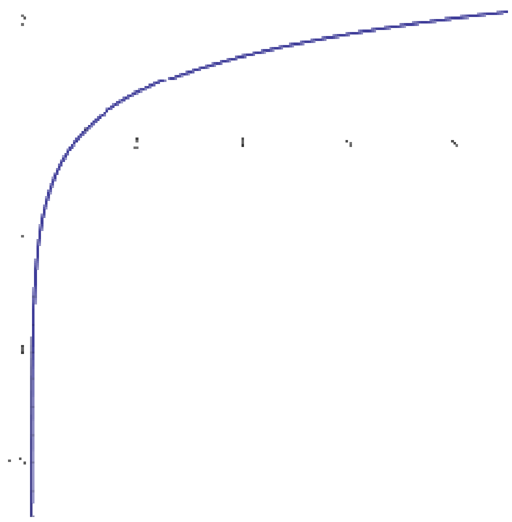
Assume $a > 0$, $a \neq 1$, so that the function $y = a^x$ is one-to-one; x also is nonnegative.

Observe:

$$a^{\log_a x} = x \text{ and } \log_a a^x = x \quad (a > 0, a \neq 1, x > 0)$$
$$e^{\ln x} = x \text{ and } \ln e^x = x \quad (x > 0)$$

The Natural Logarithm ($y = \ln x$)

- Graph:



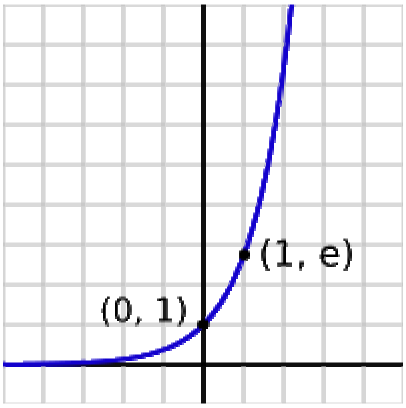
- Derivative: $d(\ln x)/dx = 1/x$

- Identities:
  $\ln ab = \ln a + \ln b$
  $\ln a/b = \ln a - \ln b$
  $\ln x^n = n \ln x$

- Logarithmic differentiation
  e.g. Use logarithmic differentiation to find the derivative of $y = (\theta \sin\theta) / \sqrt{\sec \theta}$.

The exponential Function

- Graph:



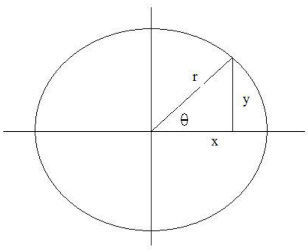
- Derivative: $d(e^x)/dx = e^x$

- Identities:

  $e = \lim_{x \to 0} (1 + x)^{1/x}$
  $a^x = e^{x \ln a}$ $(a > 0)$

Trigonometric functions and their inverses

- $360^\circ = 2\pi$ (~ 6.3 radians)



$\sin \theta = y/r$	$\csc \theta = r/y$
$\cos \theta = x/r$	$\sec \theta = r/x$
$\tan \theta = y/x$	$\cot \theta = x/y$

- graphs of trigonometric functions (Check graphs in textbook.)
- values of trigonometric functions (Check table in textbook.)
- even and odd trigonometric functions (e.g. cosine, secant are even functions; sine and tangent are odd functions.)
- Law of cosines: $c^2 = a^2 + b^2 - 2ab \cos \theta$
- Trigonometric Identities (e.g. $\cos^2 x + \sin^2 x = 1$, $\sin(x+y) = \sin x \cos y + \cos x \sin y$, etc.)
- Inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$, etc.. (check table and graphs in textbook.)
- Derivatives of trigonometric functions and their inverses, with application to integration. Some examples (also check table in textbook):

$d(\sin x)/dx = \cos x$	$d(\cos x)/dx = -\sin x$	$d(\tan x)/dx = \sec^2 x = 1 + \tan^2 x$
$d(\sin^{-1} x)/dx = 1/\sqrt{1-x^2}$	$d(\tan^{-1} x)/dx = 1/(1+x^2)$	$\int dx / (1+x^2) = \tan^{-1} x + C$

Cartesian Systems

- Cartesian coordinates $P(x,y,z)$
- vector coordinates P_1P_2 $(x_2-x_1, y_2-y_1, z_2-z_1)$ or v (v_1,v_2,v_3)
- sums of vectors $(u_1+v_1, u_2 + v_2, u_3+v_3)$
- distance between two points $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2]}$
- equation of sphere of radius a and center $P_0(x_0,y_0,z_0)$
  $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2 = a^2$

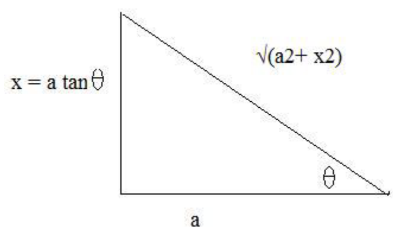
- Dot product
  It is a number, $u.v = |u||v|\cos\theta = u_1v_1 + u_2v_2+ u_3v_3$
- Projection of u onto v
  $\text{proj}_v\, u = (u.v / |v|^2)\, v$
- Cross product
  $u \times v = (|u||v|\sin\theta)\, n$ so it is a vector orthogonal to both vectors u and v.

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

- Parametrization: $r(t) = r_o + tv$, t any real number.
- Parametrization of a line passing through $P_0(x_0,y_0,z_0)$ and parallel to (v_1,v_2,v_3)
  $x = x_0 + tv_1$
  $y = y_0 + tv_2$
  $z = z_0 + tv_3$
- Equation of a plane containing P_0 and normal to vector (A,B,C)
  $A(x-x_0) + B(y-y_0) + C(z-z_0) = 0$

Integration

- Basic Integration formulas and integration by substitution
  e.g. Use integration by substitution to find $\int x^2 \sqrt{(1 + 2x^3)}\, dx$ and $\int [\cos(\ln y)/y]\, dy$.
- Integration by parts
  e.g. Use integration by parts to find $\int x \sin(x/2)\, dx$ and $\int \tan^{-1}y\, dy$.
- Partial fractions
  e.g. $(6x+7)/[(x+2)(x+3)] = A / (x+2) + B / (x+3)$; find A and B.
  also $(5x-3)/(x+2)^2 = A / (x+2) + B / (x+2)^2$; find A and B.
- Trigonometric substitutions
  e.g. $x = a \tan\theta$ allows us to replace
  $\sqrt{(a^2+ x^2)} = a \sec\theta$
  $dx = a \sec^2\theta\, d\theta$



- L'Hopital's Rule (we will learn a better method using series in MATH201):

  If $f(a) = g(a) = 0$, $f'(a)$ and $g'(a)$ exist, and $g'(a) \neq 0$
  then $\lim_{x\to a} f(x)/g(x) = f'(a)/g'(a)$.
  e.g. Find $\lim_{x\to 0} [\sqrt{(1+x)} - 1] / x$.