16

Problem 1. **[20pts]** Consider a LTI system described by the following difference equation:

$$x[n] \rightarrow \boxed{y(n) = ay(n-1) + x(n)} \rightarrow y[n]$$

Where *a* is a constant.

a) *[10pts]* Assume that the input signal $\mathbf{x(n)}$ is bounded in amplitude, that is, $|x(n)| < M < \infty$ *for all* $n \geq 0$. Determine the condition that a should satisfy in order for this system to be stable.

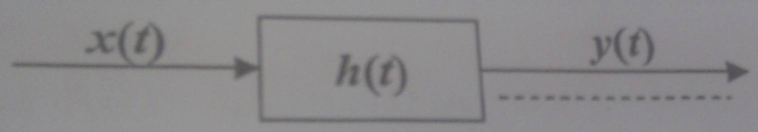
b) *[10pts]* Determine the complementary (homogeneous) solution $\mathbf{y_c(n)}$ given the initial condition $\mathbf{y(-1) = 1}$.

c) *[10pts]* Determine the particular solution $\mathbf{y_p(n)}$ when the input $\mathbf{x(n)}$ is a unit step sequence, that is, $\mathbf{x(n)=u(n)}$.

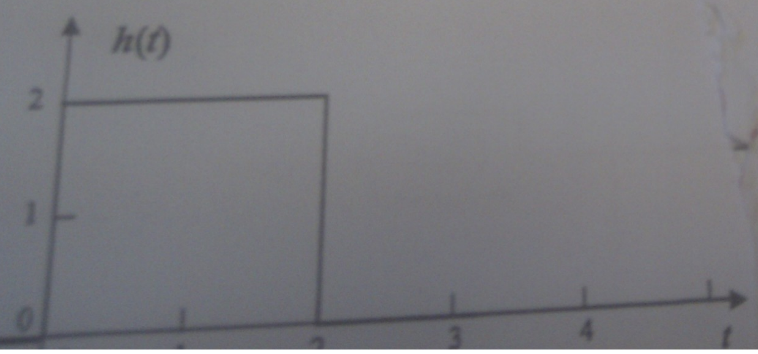
a) If x(n) is bounded then y(n) is bound

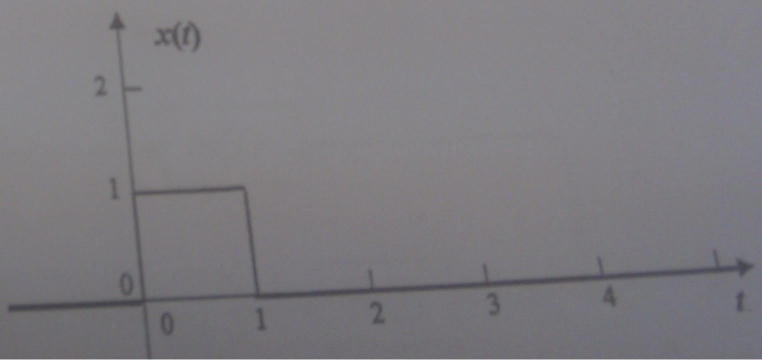
Problem 2. **[15pts]** Using the graphical convolution technique, determine and sketch the output, $y(t)$, of the following system:

15



where the input, $x(t)$ and the ~~impulse response~~, $h(t)$, are given as follows:





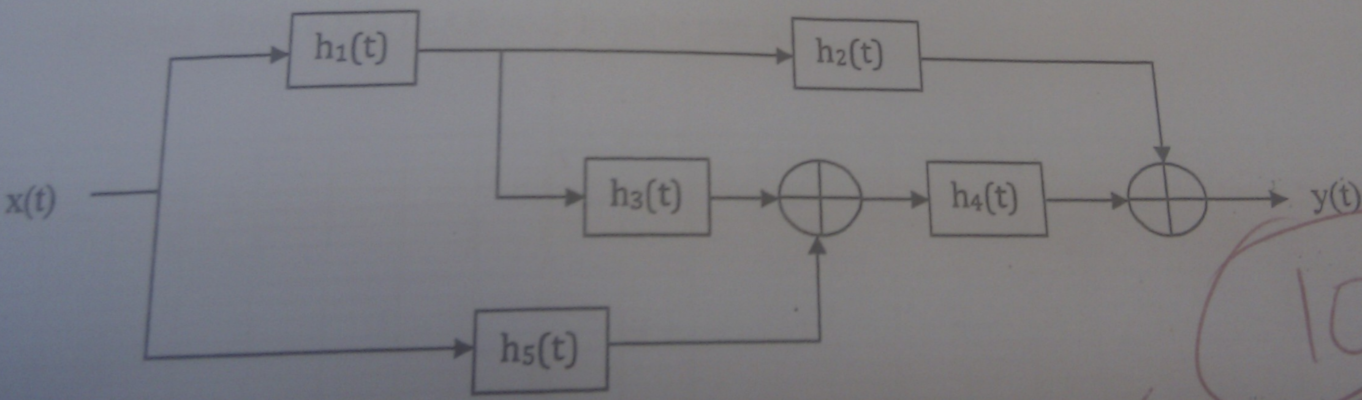
Problem 3. **[15pts]** Consider the LTI system of Figure 1 below.

15

a) [3pts] Express the system impulse response as a function of the impulse responses of the subsystems.

b) [7pts] Let $h_1(t) = h_4(t) = u(t)$ and $h_2(t) = h_3(t) = 5\delta(t)$, $h_5(t) = e^{-2t}u(t)$

Find the impulse response of the system



10

Figure 1

Problem 4. [30pts]

30

a) [2pts] Find the transfer function for the difference equation:

$$y(n) - 1.7y(n-1) + 0.72y(n-2) = x(n)$$

b) [8pts] Use the transfer function to find the steady-state response of this system for the excitation $x(n) = u(n)$.

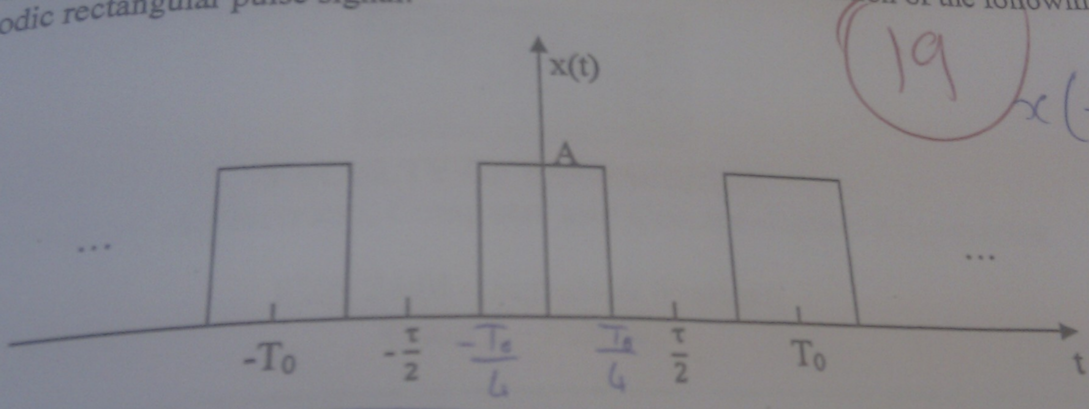
c) [8pts] Use the transfer function to find the steady-state response of this system for the excitation $x(n) = \cos(n)u(n)$.

d) [8pts] Write a MATLAB code to solve part c.

a) $H(z) = \dfrac{1}{1 - 1.7z^{-1} + 0.72z^{-2}} = \dfrac{z^2}{z^2 - 1.7z + 0.72}$

2

Problem 5. **[20pts]** Determine the Fourier series approximation of the following periodic rectangular pulse signal:



19

$$x(t) = \begin{cases} A & -\frac{T_0}{4} < t < \frac{T_0}{4} \\ 0 & \frac{T_0}{4} < t < \end{cases}$$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{jk\omega_0 t}$$

$$C_k = C_{-k}^{*}$$

$$T_0 = \frac{2\pi}{\omega_0}$$