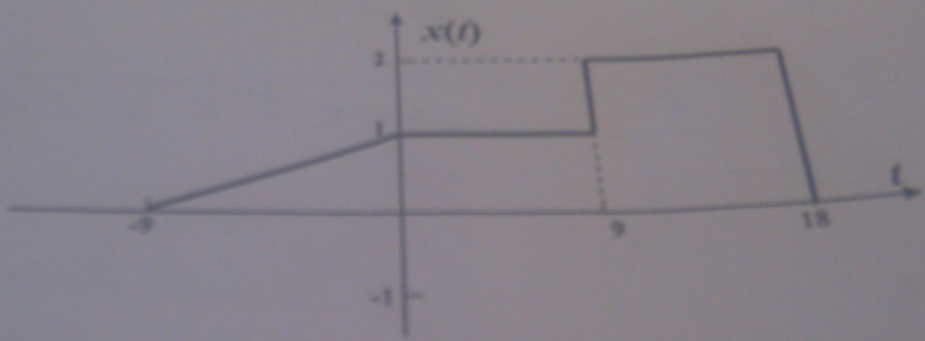
20

Problem 1. **[20pts]** Consider the following signal:



Sketch the following signals:

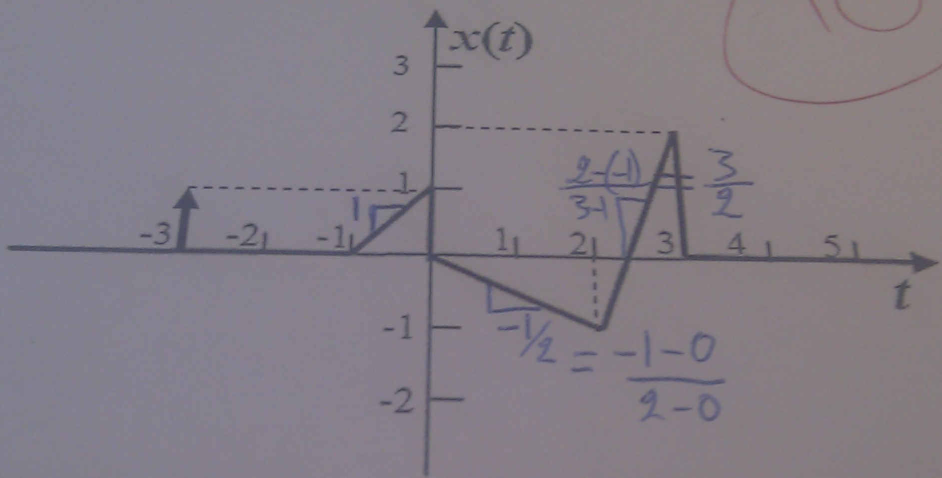
a) [10pts] $x_{even}(t)$

b) [10pts] $x_{odd}(t)$

a) $x_{even}(t) = x(t) + x(-t)$

10

Problem 2. **[20pts]** Consider the following signal:



Find the analytical expression for $x(t)$ using

a) [10pts] the method of "steps + change of slopes"

b) [10pts] the method of "windowing"

a) $x(t) = \delta(t+3) +$

21

Problem 3. **[25pts]** Find out whether each of the following signals is periodic or not (justify/clarify your answers). For each signal, determine the appropriate period or the number of samples per cycle if applicable:

a) [3pts] $v(t) = \cos 5t + 3\sin(3t + 45^{\circ})$

b) [7pts] $x(t) = \cos 200\pi t + \cos 202\pi t$ Find the maximum value of $x(t)$ in this case

c) [5pts] $v(t) = \cos t + \cos 2\pi t$

d) [5pts] $v(t) = \cos t + \cos 2pt$ *where* $p = 3.14$

e) [5pts] $x_c[n]$ sampled from $\cos 2\sqrt{\pi} t$ with a sampling period: $T = 0.1$ s.

a) $v(t) = \cos 5t + 3\sin(3t + 45°)$

Problem 4. **[10pts]** Consider the following circuit:

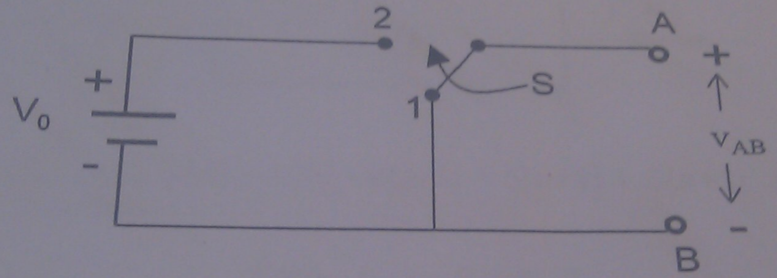
$\mu = 0$

$\mu = 1$

1

a

8



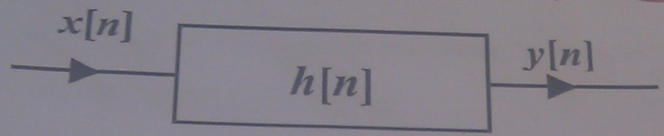
a) [2pts] The switch S in this circuit is moved to position 2 at $t=t_0$, express v_{AB} in terms of V_0 using the unit step function.

b) [8pts] If the switch S in this circuit is moved to position 2 at $t=0$ and then moved back to position 1 at $t=10$ s, express v_{AB} in terms of V_0 using the unit step function.

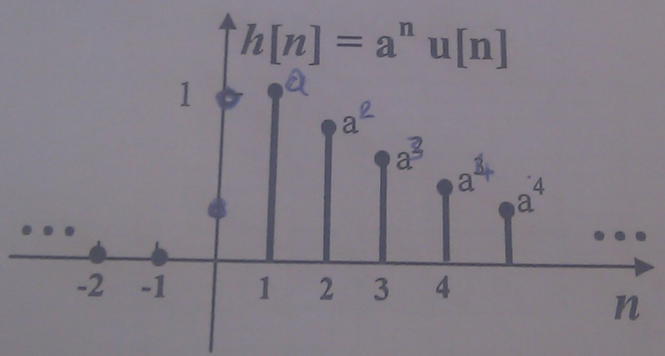
a) $V_{AB} = \cancel{V_0 u(t)}\ V_0\, u(t-t_0)$ 2

Problem 5. [25pts] Consider the following discrete-time linear, time-invariant (LTI) system:

(7)



Where the input is given by $x[n]$ = u[n] - u[n-3] + δ[n+1] + δ[n+2], and the impulse response, *is* given as follows:



a) [15pts] Determine the equation of the output, $y[n]$. Sketch the output.

Problem 5. (continued)

b) [4pts] Determine the range of values of the parameter a for which the system is **stable.**

c) [2pts] Is this system **linear** ? Provide a proof

d) [2pts] Is this system **time-invariant** ? Provide a proof

e) [1pts] Is this system **causal** ? Provide a proof

f) [1pts] Is this system **Memoryless** ? Provide a proof

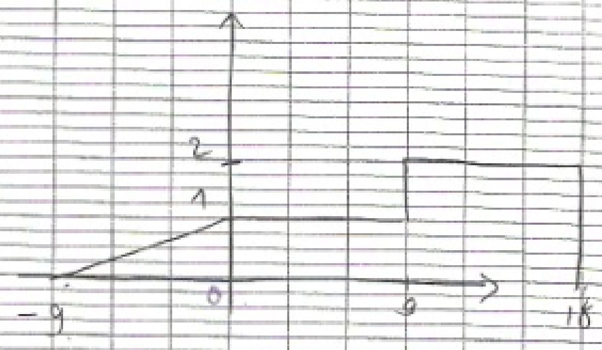
b)

0

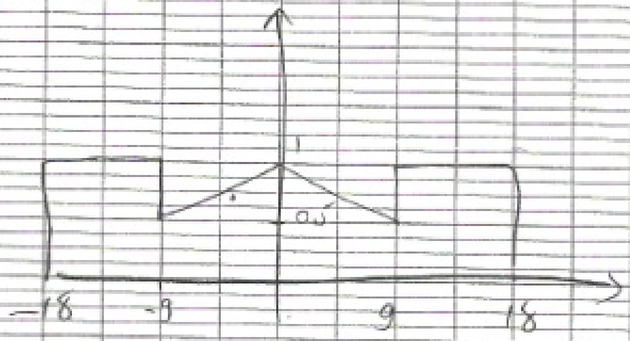
26/11/10

Solution Exam I

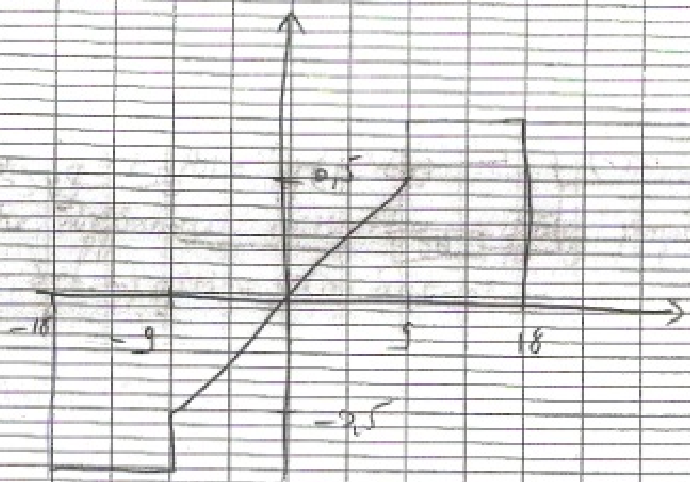
Problem I:



even

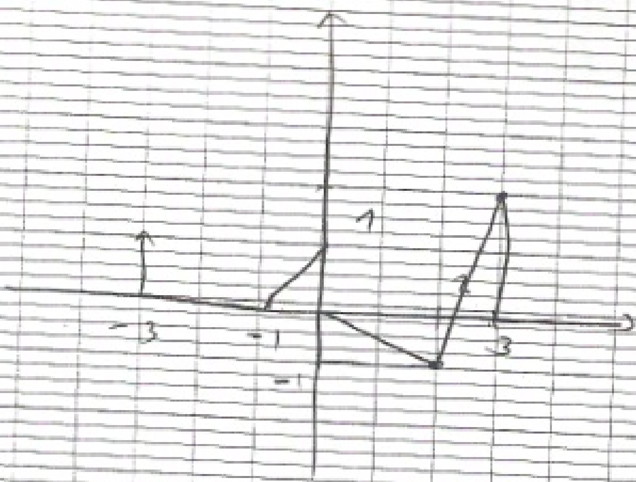


odd



Problem II

a-



a) $x(t) = \delta(t+3) + (1-0)(t+1)u(t+1) - u(t)$
$+ (0-1)\, t\, u(t) + (-\frac{1}{2}-0)\, t\, u(t) + (3-(-\frac{1}{2}))$
$(t-2)\, u(t-2)$
$- 2u(t-3) + (0-3)(t-3)\, u(t-3)$

b) $x(t) = \delta(t+3) + (t+1)[u(t+1) - u(t)]$
$- \frac{t}{2}[u(t) - u(t-2)] + (3t-7)[u(t-2)$
$- u(t-3)]$

Problem III

a) $x(t) = \cos 5t + 3\sin(3t + 45°)$

$T_1 = \frac{2\pi}{5}$ $T_2 = \frac{2\pi}{3}$ $\frac{T_1}{T_2} = \frac{3}{5} \Rightarrow$ periodic
period $= 2\pi$

b) $\cos 200\pi t + \cos 202\pi t$
$T_1 = \frac{1}{100}$; $T_2 = \frac{1}{101} \Rightarrow \frac{T_1}{T_2} = \frac{101}{100} \Rightarrow$ periodic period $= 1$
$T = 100 T_1 = 101 T$
$= 1$

c) $\cos t + \cos 2\pi t$

$T_1 = 2\pi$; $T_2 = 1 \Rightarrow$ Not periodic $V_{max} = 2$

d) $\cos t + \cos 2pt \; / \; p = 3.14$

$T_1 = 2\pi \quad T_2 = \frac{\pi}{3.14} \Rightarrow \frac{T_1}{T_2} = 6.28 = \frac{628}{100}$

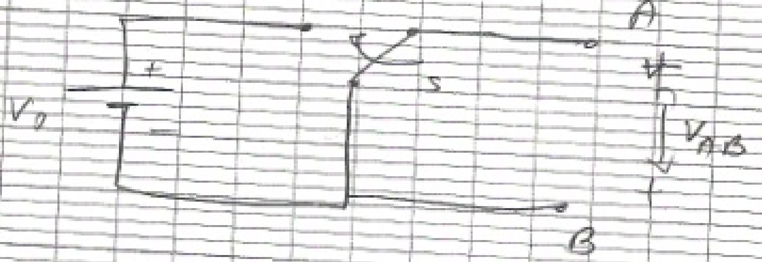
$\Rightarrow$ periodic $\quad T_0 = n_1 T_1 = n_2 T_2 = 50\pi$

period $= 50\pi$

e) $\cos \sqrt{2\pi}\, t \,/\, t = 0.1 \Rightarrow T_0 = \frac{2\pi}{\sqrt{2\pi}} = \pi \Rightarrow$ Not periodic

$\frac{T}{T_0} = \frac{0.1}{\pi}$

Problem IV



a) $V_{AB} = V_0 \, u(t - t_0)$

b) $V_{AB} = V_0 \left[u(t) - u(t - t_0) \right]$

Problem V

$x(n) = u(n) - u(n-3) + \delta(n+1) + \delta(n+2)$

$y(n) = \sum_{-\infty}^{+\infty} h(K)\, x(n-K)$

• $x(n) = u(n) \Rightarrow y_1 = \sum_{-\infty}^{+\infty} u(n-K)\, a^K u(K)$

$= \sum_{0}^{n} a^K = \frac{1 - a^{n+1}}{1-a}$

• $x(n) = u(n-3) \Rightarrow y_2 = \sum_{-\infty}^{+\infty} u(n-3-K)\, a^K u(K)$

$= \sum_{0}^{n+3} a^K = \frac{1 - a^{n+3+1}}{1-a}$

$\cdot\ x(n) = \delta(n+1) \Rightarrow y_3 = \sum_{-\infty}^{+\infty} \delta(n-k+1)\, a^k u(k)$

$= \sum_{-\infty}^{+\infty} \delta\big(-(k-n-1)\big)\, a^k u(k)$

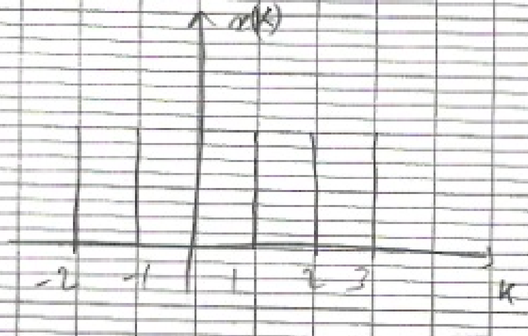
$= \sum_{-\infty}^{+\infty} \delta(k-(n+1))\, a^k u(k) \qquad \delta(-t) = \delta$

$= a^{n+1} u(n+1) \qquad \sum \delta(t-t_0) f(n) = f$

$\cdot\ x(n) = \delta(n+2)$

$$y = a^{n+2} u(n+2)$$

$$\Rightarrow \boxed{y = \frac{1-a^{n+1}}{1-a} + \frac{1-a^{n+4}}{1-a} + a^{n+1} u(n+1) + a^{n+2} u(n+}$$



$n \leq -4 \Rightarrow y(n) = 0$

$-3 \leq n \leq 3 \Rightarrow y(n) = \sum_{-3}^{} a^k = \frac{1-a^{n+1}}{1-a}$

$$y(n) = \sum_{n-2}^{n+3} a^k$$