EEN 340
SIGNALS
& TRANSFORMS

TEST 2
75 MINUTES

NDU
EE&CCE DEPARTMENT

NAME--

NOTE1: OPEN BOOK, OPEN NOTES, CLOSED NEIGHBOURS.
NOTE2: SHOW ALL WORK IN ORDER TO GET FULL CREDIT.
NOTE3: START EACH PROBLEM ON A NEW PAGE.

1. Pts. Find and sketch the frequency spectrum of the half-wave and full-wave rectified cosine waveforms shown in Fig.P1. (Hint: Use equation 5.38 in your book.)

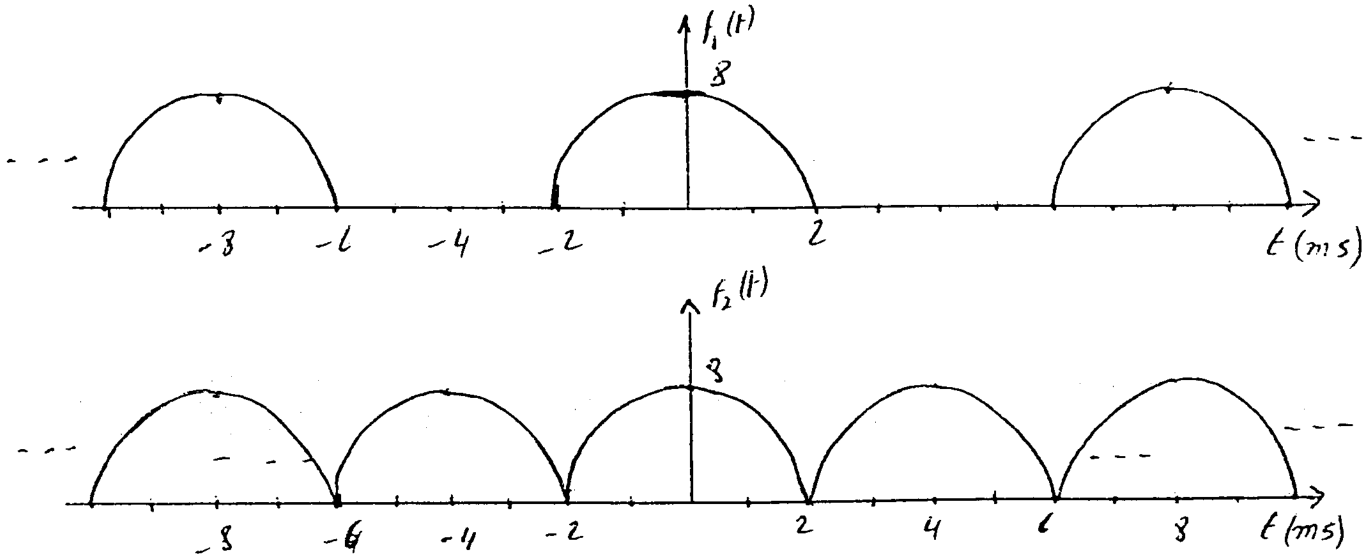


Fig.P1

2. Pts. What percentage of the total energy in the energy signal $f(t) = e^{-t}u(t)$ is contained in the frequency band -7 rad/s $\leqslant w \leqslant 7$ rad/s?

3. 20 Pts. Determine the Fourier series coefficients for the following discrete-time periodic signal. Plot the magnitude and phase of the set of coefficients a_k.

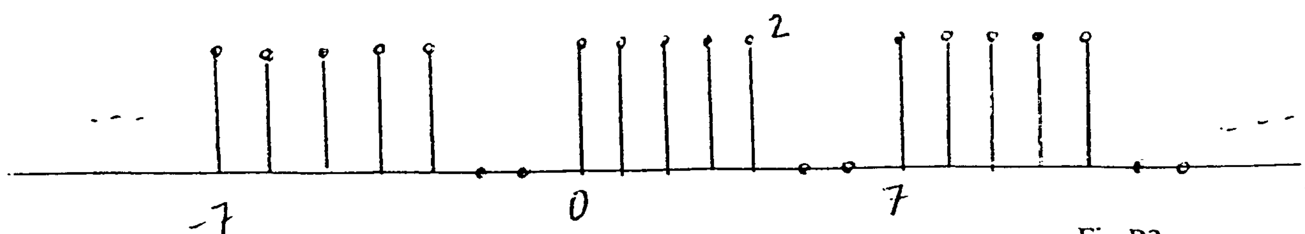


Fig.P3.

4. 20 Pts. Let x(t) be a periodic signal with fundamental period T and Fourier series coefficients a_k. Derive the Fourier series coefficients of each of the following signals in terms of a_k:

   a. even{x(t)}
   b. $d^2x(t)/dt^2$

5. 20 Pts. The input and the output of a stable and causal LTI system are related by the differential equation

   $$d^2 y(t)/dt^2 + 6\, dy(t)/dt + 8\, y(t) = 2x(t)$$

   a. Find the impulse response of this system.
   b. What is the response of this system if $x(t) = te^{-2t}u(t)$?

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$$

~~Question~~ Solution

... (Test 2)

2) $f(t) = e^{-t} u(t) \longleftrightarrow \frac{1}{1+j\omega}$

$|F(\omega)|^2 = \frac{1}{1+\omega^2}$

$E_T = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{1+\omega^2} d\omega = \frac{1}{\pi} \int_{0}^{\infty} \frac{1}{1+\omega^2} d\omega$

$E_T = \frac{1}{\pi} \tan^{-1}(\omega) \Big|_0^{\infty} = \frac{1}{\pi} \frac{\pi}{2} = \boxed{\frac{1}{2}\ J}$

For $-7 \leq \omega \leq 7$

$E_T = \frac{1}{\pi} \int_0^{7} \frac{1}{1+\omega^2} d\omega = \frac{1}{\pi} \tan^{-1}(\omega)\Big|_0^{7} = \frac{1}{\pi}\left(\frac{\pi}{2}\right) 0.909 = 0.4548\ J$

$\% = \frac{0.4548}{0.5} \times 100 = \boxed{91\ \%}$

b) Let $x(t) \longrightarrow a_k$

even $\{x(t)\} = \frac{1}{2}\left(x(t) + x(-t)\right)$

$x(t) \longrightarrow a_k$

$x(-t) \longrightarrow b_k = \frac{1}{T} \int x(t)\, e^{-j(k 2\pi/T) t} dt = a_{-k}$

$\boxed{Ev\{x(t)\} \longrightarrow C_k = \frac{1}{2}\left(a_k + a_{-k}\right)}$

$x(t) \longrightarrow a_k \qquad \frac{d^2 x(t)}{dt^2} \longrightarrow b_k \qquad x(t) = \sum a_k e^{j\left(\frac{2\pi}{T}\right)}$

$x(t) = \sum a_k e^{j\omega_0 k}$

$\frac{dx(t)}{dt} = \sum j\omega_0 k a_k e^{jk\omega_0 t}$

$\frac{d^2 x(t)}{dt^2} = \sum j^2 \omega_0^2 k^2 a_k e^{jk\omega_0 t} = \sum -k^2 \omega_0^2 a_k e^{jk\omega_0 t}$

$$\boxed{\frac{d^2 x(t)}{dt^2} \longrightarrow b_k = -k^2 \frac{4\pi^2}{T^2} a_k}$$

$$\frac{d^2 y(t)}{dt} + 6\frac{dy}{dt} + 8y(t) = 2x(t)$$

$$-\omega^2 y(j\omega) + 6 j\omega y(j\omega) + 8 y(j\omega) = 2x(j\omega)$$

$$\frac{y(j\omega)}{x(j\omega)} = H(j\omega) = \frac{2}{(j\omega)^2 + 6j\omega + 8} = \frac{2}{(j\omega+2)(j\omega+4)}$$

$$= \frac{A}{j\omega+2} + \frac{B}{j\omega+4}$$

$$A = \frac{2}{j\omega+4}\bigg|_{j\omega=-2} = \frac{2}{2} = 1$$

$$B = \frac{2}{j\omega+2}\bigg|_{j\omega=-4} = -1$$

$$H(j\omega) = \frac{1}{j\omega+2} - \frac{1}{j\omega+4} \Rightarrow h(t) = e^{-2t}u(t) - e^{-4t}u(t)$$

$$x(t) = te^{-2t}u(t) \quad x(j\omega) = \frac{1}{(2+j\omega)}$$

$$y(j\omega) = H(j\omega)\,x(j\omega) = \frac{2}{(j\omega+2)^3(j\omega+4)}$$

$$= \frac{A}{(j\omega+2)^3} + \frac{B}{(j\omega+2)^2} + \frac{C}{(j\omega+2)} + \frac{D}{j\omega+4}$$

$$A = \frac{2}{j\omega+4}\bigg|_{j\omega=-2} \quad \frac{2}{2} = 1$$

$$B = -0.5 \;;\; C = \frac{1}{4} \;;\; D = -\frac{1}{4}$$

$$y(j\omega) = \frac{1}{(j\omega+2)^3} - \frac{0.5}{(j\omega+2)^2} + \frac{1/4}{j\omega+2} \quad \frac{-1/4}{j\omega+4}$$

$$y(t) = t^2 e^{-2t}u(t) - 0.5\,te^{-2t}u(t) + 0.25\,e^{-2t}u(t) - 0.25\,e^{-4t}u(t)$$

$T = 8\,ms \qquad \omega_0 = \frac{2\pi}{8} \times 10^3 = 250\pi$

$f_1(t) = \sum_{n=-\infty}^{\infty} g_1(t - nT) = \sum_{n=-\infty}^{\infty} g_1(t - n8\times10^{-3})$

$g_1(t) = 8\cos\omega_0 t \; \Pi\left(\frac{t}{4\times10^{-3}}\right) = 8\cos 250\pi t \; \Pi\left(\frac{t}{4\times10^{-3}}\right)$

$$\boxed{g_1(t) = 4\Pi\left(\frac{t}{4\times10^{-3}}\right)e^{j250\pi t} + 4\Pi\left(\frac{t}{4\times10^{-3}}\right)e^{-j250\pi t}}$$

$\Pi\left(\frac{t}{T}\right) \longrightarrow T\,\mathrm{sinc}\,T\omega \qquad x(t)e^{j\omega_0 t} \longrightarrow x(f - f_0)$

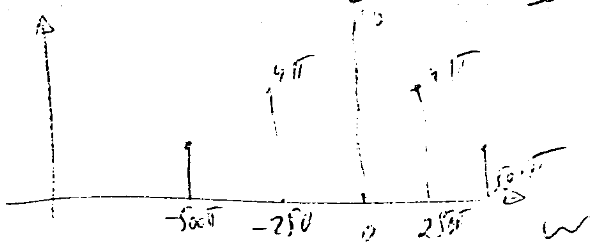
$16\times10^{-3}\,\mathrm{sinc}\;2\times10^{-3}\omega$

or $x(t)\cos\omega_0 t \longleftrightarrow \frac{1}{2}x(f-f_0) + \frac{1}{2}x(f+f_0)$

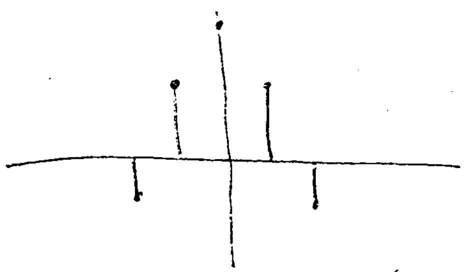
$16\times10^{-3}\left[\mathrm{sinc}(2\times10^{-3})(\omega - 250\pi)) + \mathrm{sinc}\,(2\times10^{-3})(\omega+250\pi)\right]$

$F_1(\omega) = \sum_{n=-\infty}^{\infty} \omega_0 G_1(n\omega_0)\,\delta(\omega - n\omega_0)$

$F_1(\omega) = \sum 4\pi\left[\mathrm{sinc}\frac{(n-1)}{2}\pi + \mathrm{sinc}\frac{(n+1)\pi}{2}\right]\delta(\omega - n250\pi)$



b) $F_2(\omega) = \sum_{n=-\infty}^{\infty} 8\pi\left(\mathrm{sinc}\,(2n-1)\frac{\pi}{2} + \mathrm{sinc}\left[(2n+1)\frac{\pi}{2}\right]\right)\delta(\omega - n500\pi)$



3) $a_k = \frac{2}{7}\frac{\sin(5\pi k/7)}{\sin(\pi k/7)}$

$b_k = a_k e^{-j2k\omega_0} \qquad a_k e^{-j\frac{2\cdot2\pi}{7}k} \qquad a_k e^{-j\frac{4\pi k}{7}}$

$b_k = \frac{2}{7}e^{-j4\pi\frac{k}{7}}\frac{\sin(5\pi k/7)}{\sin(\pi k/7)} \qquad n \neq 0, \mp7, \mp14$