EE 340
SIGNALS AND TRANSFORMS

TEST 2
50 MINUTES

NDU
EE DEPARTMENT

NOTE1: OPEN BOOK, OPEN NOTES, CLOSED NEIGHBOURS.
NOTE2: SHOW ALL WORK IN ORDER TO RECEIVE FULL CREDIT.

1. 15 Pts. Draw the direct form II block diagram of the system equation below.

$$dy(t)/dt = x(t) + 2dx(t)/dt + 3\int_{-\infty}^{t} x(\tau)\, d\tau$$

2. 20 Pts. Determine the Fourier series representation for the following signal shown in Fig.P2.

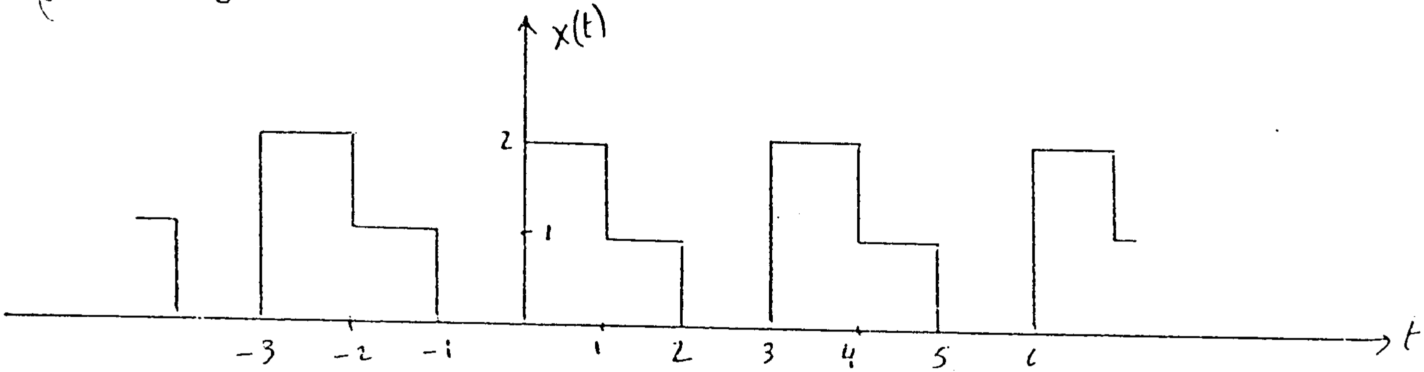

Fig.P2.

3. 20 Pts. Find the Fourier series coefficients for the signal shown in Fig. P3. Shown below.

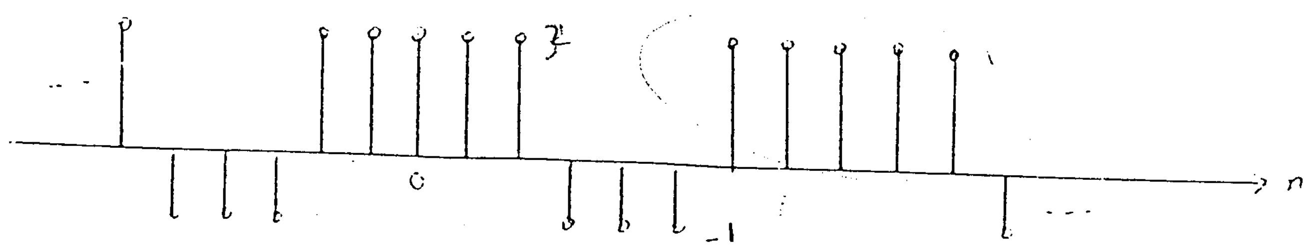

Fig.P3.

4. 20 Pts. Compute the convolution of each of the following pairs of signals x(t) and h(t) by calculating X(jw) and H(jw), using the convolution property, and inverse transforming.

$$x(t) = te^{-2t}u(t), \quad h(t) = te^{-5t}u(t)$$

$\frac{3}{N}$ sin $2\pi k/$ $a_k = \frac{2 \cdots}{N} = \frac{5}{8}$

5. 25 Pts. Compute the Fourier transform of the following signal shown in Fig.P5.

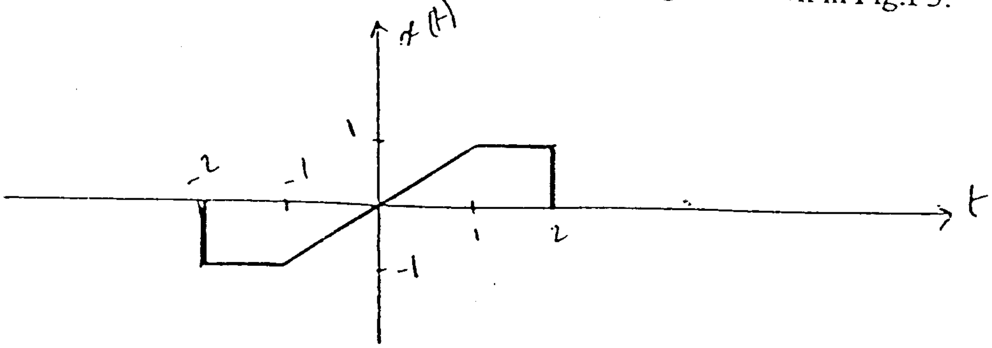

Fig.P5.

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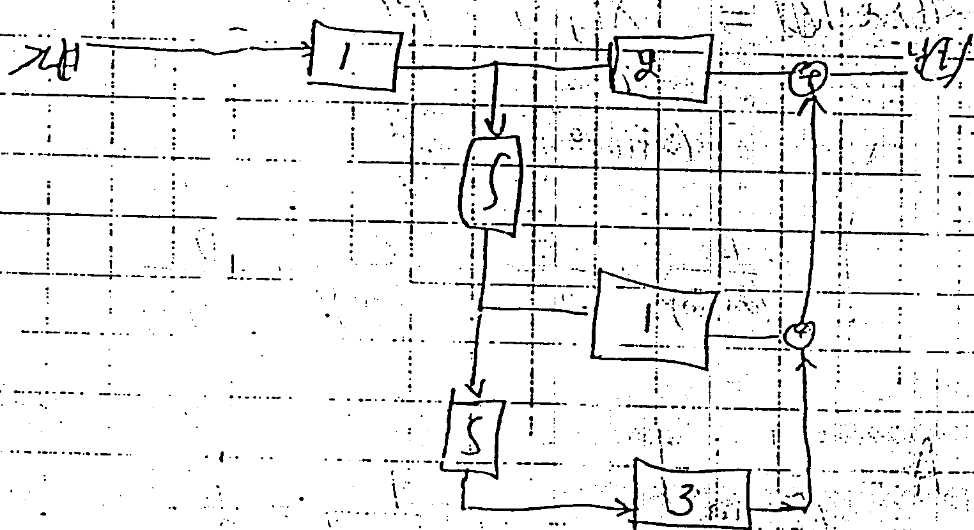
$$\frac{dy(t)}{dt} = x(t) + 2\frac{dx}{dt} + 3\int_{-\infty}^{t} x(\tau)d\tau$$

(I) $$\frac{d^2y(t)}{dt} = \frac{dx(t)}{dt} + 2\frac{d^2x(t)}{dt} + 3x(t)$$

$a_2 = 1$, $a_1 = 0$, $a_0 = 0$

$b_2 = 2$, $b_1 = 1$, $b_0 = 3$



4.

$x(t) = te^{-2t}u(t) \Rightarrow h(t) = te^{-5t}u(t)$

$x(t) * h(t) \longleftrightarrow X(j\omega)\cdot H(j\omega)$

$x(t) = te^{-2t} \longleftrightarrow \frac{1}{(2+j\omega)^2}$

$h(t) = te^{-5t} \longleftrightarrow \frac{1}{(5+j\omega)^2}$

$x(t) * h(t) = X(j\omega)\cdot H(j\omega)$

$= \frac{1}{(2+j\omega)^2}\cdot\frac{1}{(5+j\omega)^2} =$

$= \frac{A}{(2+j\omega)^2} + \frac{B}{(2+j\omega)} + \frac{C}{(5+j\omega)^2} + \frac{D}{(5+j\omega)}$

$A = \frac{1}{(5+j\omega)^2}\Big|_{j\omega=-2} = \frac{1}{(5-2)^2} = \frac{1}{9}$

$C = \frac{1}{(2+j\omega)^2}\Big|_{j\omega=-5} = \frac{1}{(2-5)^3} = \frac{1}{9}$

$j\omega = -4$

$\frac{1}{(2)^2(1)^2} = \frac{1/9}{(-2)^2} + \frac{B}{(-2)} + \frac{1/9}{(1)^2} + \frac{D}{1}$

$\frac{1}{4} = \frac{1}{36} - \frac{B}{2} + \frac{1}{9} + D$

$-\frac{B}{2} + D = \frac{1}{9}$

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$j\omega = 0$

$$\frac{1}{4}\cdot\frac{1}{25} = \frac{1/9}{4} + \frac{B}{2} + \frac{1/9}{25} + \frac{D}{5}$$

$$\frac{1}{100} = \frac{1}{36} + \frac{B}{2} + \frac{1}{225} + \frac{D}{5}$$

$$\frac{-1}{45} = \frac{B}{2} + \frac{D}{5}$$

$\Rightarrow$

$$\frac{B}{2} + \frac{D}{5} = \frac{-1}{45}$$

$$\frac{-B}{2} + D = \frac{1}{9}$$

$$\frac{6D}{5} = \frac{4}{45} \Rightarrow 6D = \frac{4}{9}$$

$$\Rightarrow D = \frac{4}{6\times 9} = \frac{2}{27}.$$

$$\frac{-B}{2} + \frac{2}{27} = \frac{1}{9} \Rightarrow -\frac{B}{2} = -\frac{2}{27} + \frac{1}{9}$$

$$B = -\frac{2}{27}.$$

$$X(j\omega)\cdot H(j\omega) = \frac{1/9}{(2+j\omega)^2} + \frac{-2/27}{(2+j\omega)} + \frac{1/9}{(5+j\omega)^2} + \frac{2/27}{(5+j\omega)}$$

$$x(t) * h(t) = \frac{1}{9}te^{-2t}u(t) - \frac{2}{27}e^{-2t}u(t) + \frac{1}{9}te^{-5t}u(t) + \frac{2}{27}e^{-5t}u(t).$$

$$B = \frac{d}{d\omega}\left(\frac{1}{(5+j\omega)^2}\right)\Bigg|_{j\omega=-2} = \frac{0-(-10-2j\omega)\,j}{\ldots} = \frac{-6}{\ldots} = -\frac{2}{\ldots}$$

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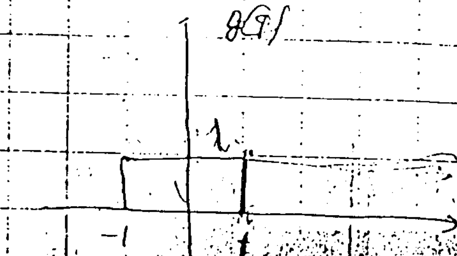
$$x(t) = \begin{cases} -1 & -2<t<-1 \\ t & -1<t<1 \\ 1 & 1<t<2 \end{cases}$$

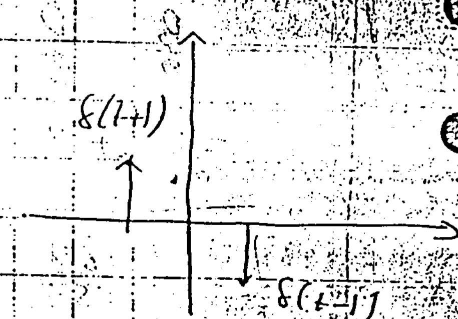
$$g(t) = \frac{dx(t)}{dt} = \begin{cases} 0 & -2\le t\le -1 \\ 1 & -1<t\le 1 \\ 0 & 1\le t<2 \end{cases}$$



$$g(t) = u(t+1) - u(t-1)$$

impulse fn

$$h(t) = \frac{dg(t)}{dt} = \delta(t+1) - \delta(t-1)$$



$$H(j\omega) = j\omega G(j\omega) = e^{+j\omega} - e^{-j\omega}$$

$$G(j\omega) = \frac{H(j\omega)}{j\omega} + k\delta(\omega)$$

$$= \frac{H(j\omega)}{j\omega} + H(0)\delta(\omega)\pi$$

The average value is zero.

$$G(j\omega) = \frac{1}{j\omega}\left(e^{+j\omega} - e^{-j\omega}\right) = \frac{2}{\omega}$$

$$G(j\omega) = j\omega X(j\omega)$$

$$\Rightarrow X(j\omega) = \frac{1}{j\omega}G(j\omega) + k\delta(\omega)$$

$$= \frac{1}{j\omega}G(j\omega) + \delta(\omega)\pi G(0)$$

$$= \frac{-1}{\omega^2}\left[e^{+j\omega} - e^{-j\omega}\right] + c$$

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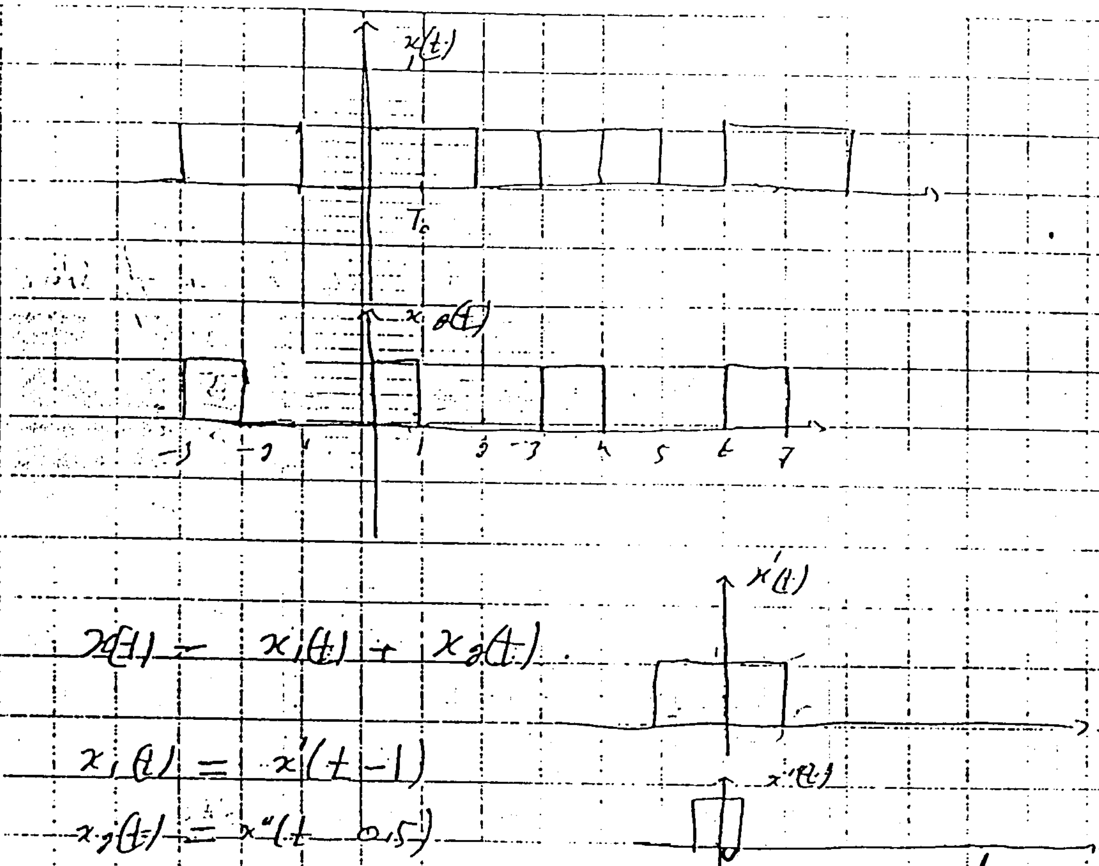

$x(t) = x_1(t) + x_2(t)$.

$x_1(t) = x'(t-1)$

$x_2(t) = x''(t-0.5)$

For $x'(t)$.

$x'(t)$: $T_1 = 1$ $T = 3$ $\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{3}$

$$a'_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T} = \frac{2\sin(k\frac{2\pi}{3})\cdot 1}{3k\frac{2\pi}{3}}$$

$$= \frac{2\sin(k\frac{2}{3})}{3 \cdot k}$$

$a'_0 = \frac{2T_1}{T} = \frac{2}{3}$.

For $x''(t)$: $T_1 = 0.5$ $T = 4$ $\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$$a''_k = \frac{2\sin(k\omega_0 T_1)}{} = \frac{2\sin(k\frac{\pi}{3})}{}$$

$$a''_0 = \frac{2T_1}{T} = \frac{2}{4(2)} = 1$$

$$x(t) = x_1(t) + x_2(t)$$

$$d_k = a'_k + a''_k = \frac{2}{3}\,\text{sinc}\left(\frac{2}{3}k\right) + \frac{1}{4}\,\text{sinc}\left(\frac{k}{4}\right)$$

$$d_0 = a'_0 + a'' = \frac{2}{3} + 1 = \frac{5}{3}$$

$$a'_k = \frac{2}{3}\,\text{sinc}\left(\frac{2k}{3}\right) \qquad a''_k = \frac{1}{3}\,\text{sinc}\left(\frac{k}{3}\right)$$

$$a'_0 = \frac{2}{3} \qquad a''_0 = \frac{1}{3}$$

$$b_k \begin{cases} b_k = \frac{2}{3}\,\text{sinc}\left(\frac{2k}{3}\right) + \frac{1}{3}\,\text{sinc}\left(\frac{k}{3}\right) & k \neq 0 \\ b_0 = \frac{2}{3} + \frac{1}{3} = 1 & k = 0 \end{cases}$$

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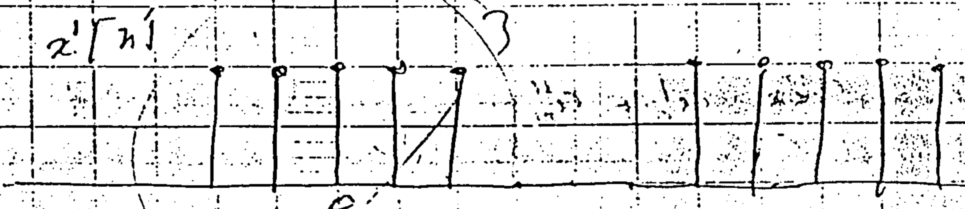
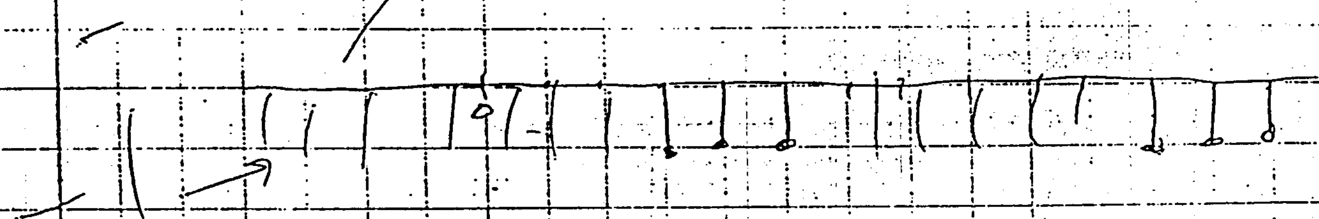
$$a_k = \frac{1}{N} \frac{\sin(2\pi k(N_1+1/2)/N)}{\sin(\pi k/N)}$$

$$a_k = \frac{2N_1+1}{N}$$

$x'[n]$



$x''[n]$



$$x[n] = x'[n] + x''[n]$$

Where $x'[n]$ $\quad N = 8 \quad N_1 = 2.$

$$a_k = \frac{1}{8} \frac{\sin(2\pi k(\frac{5}{16}))}{\sin(\frac{\pi k}{8})} \qquad \frac{1}{8} \frac{\sin\frac{5k\pi}{8}}{\sin\frac{\pi k}{8}} =$$

$$a_0 = \frac{4+1}{8} = \frac{5}{8}$$

4 $\quad x''[n] = -x[n-3] \quad N = 8 \quad N_1 = 1$

$a_0'' = \frac{8(1)+1}{8} = \frac{3}{8}$

$d_k = \left\{ \frac{\frac{1}{8}\sin\left(2\pi k\left(\frac{5}{16}\right)\right)}{\sin\left(\frac{\pi k}{8}\right)} \right. \quad \frac{\frac{1}{8}\sin\left(\frac{3}{8}\pi k\right)}{\sin}$

$a_k = a_0' + a_{01}''$

$a_k = \frac{15}{8} - 1 = \frac{7}{8}$