

NOTE 1: OPEN BOOK, OPEN NOTES.

NOTE 2: SHOW ALL WORK IN ORDER TO RECEIVE FULL CREDIT.

NOTE 3: START EACH PROBLEM ON A NEW PAGE.

1. 30 Pts. A causal LTI system S with impulse response $h(t)$ has its input $x(t)$ and output $y(t)$ related through a linear constant-coefficient differential equation of the form

$$\frac{d^3 y(t)}{dt^3} + (1 + \alpha) \frac{d^2 y(t)}{dt^2} + \alpha(\alpha + 1) \frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$$

(a) If

$$g(t) = \frac{dh(t)}{dt} + h(t)$$

how many poles does $G(s)$ have?

(b) For what real values of the parameter α is S guaranteed to be stable?

2. 35 Pts. Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{d^2 y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)$$

Let $X(s)$ and $Y(s)$ denote Laplace transforms of $x(t)$ and $y(t)$, respectively, and let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response.

- (a) Determine $H(s)$ as a ratio of two polynomials in s . Sketch the pole-zero pattern of $H(s)$.
- (b) Determine $h(t)$ for each of the following cases:
1. The system is stable.
 2. The system is casual
 3. The system is *neither* stable *nor* causal.

3. 35 Pts. The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}$$

Determine and sketch the response $y(t)$ when the input is

$$x(t) = e^{-t}, \quad -\infty < t < \infty$$